Subsistence Consumption, Investment, and Poverty

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Abstract

This paper develops an endogenous growth model in which the extent and depth of poverty influence the aggregate outcomes of the economy. Individuals are bound by a subsistence consumption requirement and their risk aversion depends negatively on their distance from the constraint. Agents divide their time between farming and working for a wage. Farm productivity can be improved stochastically through household investment. The impact that subsistence consumption has on household investment and economic growth is decomposed into constraints on disposable income and increased risk aversion. Constraints on disposable income determine the household's innovation decision when they are extremely poor. However, as incomes rise to levels similar to that of India in the 1980s increased risk aversions starts to dominate household choices. Quantitative work suggests that the subsistence constraint can reduce the annualized growth rate by as much as 65 basis points. Consistent with recent empirical studies, conditional cash transfers out-perform unconditional cash transfers in terms of growth outcomes, due to their creation of incentives to invest in higher risk activities.

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1 Introduction

Economic development is characterized by the process of structural change, with people moving from rural communities to urban centers and from agriculture production to manufacturing employment. Extensive research on structural change has identified a common constraint to this process, the efficiency of agricultural production. Schultz (1953) referred to this constraint as the food problem, a society must be able to feed itself before it can move on to producing manufactured and service goods. This is particularly challenging for developing countries, because even though a large share of their population is employed in agriculture and other subsistence industries, these industries are often characterized by low productivity.¹

Broadly speaking, the food problem can be thought of as one of trapped factors, in which low productivity in the agriculture sector results in an inability to allocate resources to the manufacturing and service sectors. In this context, the problem of structural change can be reinterpreted as one of productivity growth in subsistence sectors. For the most part, the literature on structural change has focused on constraints that are external to the individual: inappropriate technologies, high costs, distribution of resources, etc. But in recent years there has been a concerted effort (especially in agriculture) to provide individuals in developing countries with affordable access to appropriate technologies and yet the adoption has been slow, which would imply that the constraints to structural change are not completely external. This paper focuses on one potential internal constraint: the consequences of poverty as they relate to risk-taking. Specifically, I look at how increased risk aversion that results from being close to one’s subsistence requirement results in slower adoption of technology. Intuitively, negative income shocks are particularly hard on those agents who are near subsistence. A loss of income will result in a reduction of already low levels of consumption. The effects of this reduction could range from low productivity due to malnutrition to, in the case of prolonged exposure, outright starvation. This would result in lower risk taking, which is problematic if those activities that are growth enhancing are also risky.

This paper starts with the premise that in developing countries a significant share of the population is in relatively close proximity to its subsistence requirements. Households are endowed with preferences that exhibit decreasing relative risk aversion (DRRA) and must satisfy their subsistence constraint at all times. They split their unit time endowment between farming and working for a wage. Farm productivity can be improved stochastically through household investment. This paper shows that the household’s proximity to their subsistence constraint di-

¹Restuccia et. al. (2008) shows that labor productivity is extremely low in developing countries. GDP per worker for the agricultural sector in the richest countries is 78 times higher than that in developing countries.
rectly impacts their willingness to innovate.

This paper makes three contributions. The first contribution deals with the macroeconomic consequences of subsistence consumption. Steger (2000) and Chatterjee and Ravikumar (1999) both show that subsistence consumption has negative effects on economic growth. However, both of these models focus on asset accumulation and the growth effects thereof. To the best of the author’s knowledge this paper represents the first to look at the interaction of risk and subsistence consumption and their effects on economic growth. 2 In particular, it shows that risk aversion amplifies the effect of subsistence consumption found in Steger (2000) and Chatterjee and Ravikumar (1999). The reason for this amplification is that where the previous two papers focused on the accumulation of safe assets as the engine of growth, this paper considers risky investment in order to drive forward productivity. Donovan (2015) uses subsistence consumption and exogenous idiosyncratic productivity shocks to examine cross-country agricultural productivities. In contrast to Donovan, this paper looks at growth rates, and not only allows agents to choose their risk exposure, but to avoid it completely if they so desire.

The second contribution deals with how poor households make decisions regarding risky investments. The inclusion of the subsistence constraint not only results in increased risk aversion among the poor, but it also imposes a constraint on disposable income. Households ultimately only have discretion over their income net of subsistence. There have been several empirical and experimental studies which have documented household investment in developing countries. The most relevant being, Bandiera and Rasul (2006), who relate risk aversion directly to subsistence consumption, showing that when presented with the opportunity to produce a new cash crop, households that were more food secure were also more likely to adopt that crop. However, it is difficult to ascertain how much of these investment decisions are due to constraints on disposable income and how much are the result of increased risk aversion. This paper adds to the literature by decomposing the effects of the disposable income and risk aversion constraints on the household's decision. In particular, the numerical results presented later in the paper show that the constraints on disposable income have the greatest effect when the economy is extremely poor, however throughout the process of development this constraint quickly dissipates and households become more concerned with the risk of investment.

The final contribution deals with the policy implications of the model. Given the primary focus on the food security of households the most reasonable policy to consider is cash transfers. There is a vast experimental literature on cash transfers showing their (nearly) uniformly pos-

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2Donovan (2016) does model the interaction between risk and subsistence consumption, however, he focuses on cross country productivity differences rather than economic growth.
As vast and positive as the literature is, there is still significant debate over the optimal form of these cash transfers. This paper contributes to this debate by using the model presented in this paper to examine the long-run effects of both conditional and unconditional cash transfers. Consistent with the existing literature, both types of cash transfers result in increased investment and greater food security. However, conditional transfers significantly outperform unconditional by several orders of magnitude.

In addition to the papers listed above, this paper is related to three literatures: risk-taking, barriers to technological adoption and structural change. There is substantial empirical and experimental evidence that risk aversion associated with low levels of income can result in a slow adoption of new technologies. This was evidenced with new crop adoption during the Green Revolution. The technologies introduced during the Green Revolution were associated with significant increases in productivity (Evenson and Gollin, 2003). Even so, the rate of adoption was far from uniform. Munshi (2004) and BenYishay and Mobarak (2013) show that the rates of adoption were highest in relatively homogenous communities. They found that the key determinant in the adoption of a new variety of crop was the ability to learn from the experiences of others. Social learning in the presence of homogeneous conditions is an important determinant in lowering idiosyncratic risk. Under relatively uniform conditions, the successful implementation of a new technology by one member of a community will most likely be indicative of the experience the other members. For example, Munshi (2004) showed that the adoption of high-yield crops was greater in northern India where growing conditions were far more uniform than those in southern India.

Sadler (2000) finds that poverty traps are eliminated in the presence of a risk taking technology. This paper builds upon Sadler’s model by adding a subsistence consumption constraint so that households exhibit decreasing relative risk aversion. Banerjee and Duflo (2007) andBinswanger and Rosenzweig (1993) both show that the poor engage in activities that limit their exposure to risk at the cost of lowering their incomes. On the macro side, Acemoglu and Zilibotti (1997) consider how an inability to completely diversify risk affects economic growth. They use indivisible projects that keep agents from diversifying to show that market-incompleteness can hinder capital accumulation and growth.

Several barriers to adoption have been identified in the literature including: education (Nelson and Phelps, 1966, and Caselli, 1999), political resistance (Parente and Prescott, 1994), and inappropriate technologies (Atkinson and Stiglitz, 1969 and Acemoglu and Zilibotti, 2001). This research adds to the literature by including factors stemming from risk aversion close to subsistence. The last related literature is that of structural change. Chanda and Dalgaard (2008) show

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that the relative efficiency between two sectors is determined by constraints on the distribution of resources and the relative level of technology. Both are features of the model in this paper. This paper is also related to those papers that consider the transition from the Malthusian growth regime to a modern one.⁴ That literature, as well as this paper, considers the transition from traditional production, low productivity methods to modern production.

This paper proceeds as follows: section 2 presents the full theoretical model. Section 3 looks at a simple version of the theoretical model to generate analytical results and build intuition. Computational results are found in section 4, section 5 looks at the relative efficiency of conditional and unconditional cash transfers, and section 6 concludes.

## 2 A Simple Model

### 2.1 Set-up

Suppose the economy is populated with a continuum of households on the unit measure. Each household is active for one period and must consume enough of the unique final good to satisfy their subsistence requirement. Utility from consumption is net of subsistence, such that household preferences are given by:

\[
U(c_{it}) = E_t \log(c_{it} - \bar{c})
\]

where \(c_{it}\) is household consumption and \(\bar{c}\) is their subsistence requirement. Note that the inclusion of a subsistence requirement causes households to exhibit decreasing relative risk aversion (DRRA). The resulting measures of absolute and relative risk aversion are: \(r_A(c) = 1/(c - \bar{c})\) and \(r_R(c) = c/(c - \bar{c})\), respectively. This means that an increase in a household’s poverty (as measured by their distance from the subsistence requirement) will result in a greater degree of risk aversion even when the gamble is proportional to their income.⁵

Each household acts as a yeoman-farmer producing the final good \((y_{it})\) with technology \(y_{it} = q_{it}\), where \(q_{it}\) their is productivity. Total output in the economy is:

\[
Y_t = \int_0^1 q_{it} di
\]

Households can attempt to improve their productivity stochastically by investing \(z_{it}\) units of the

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⁴For examples of this literature see Galor and Weil (2000).

⁵Decreasing relative risk aversion is consistent with the evidence presented by Ogaki and Zhang (2001).
final good. Successful innovation (with exogenous probability $\chi$) results in an increase in household productivity, while failure is met with a reduction.\footnote{Intuitively this could be thought of as a farmer experimenting with a new variety of crop by dedicating a portion of his fields to it. Failure to adopt this new crop would lead to a loss of output.} Formally, the evolution of household productivity is:

$$q_{it} = \begin{cases} q_{i(t-1)}(1 + z_{it}) & \text{w/ probability } \chi \\ q_{i(t-1)}(1 - z_{it}) & \text{w/ probability } 1 - \chi \end{cases}$$

The entirety of production is consumed by the household, so that the budget constraint is:

$$c_{it} = y_{it} = q_{it}$$

### 2.2 Household Decision

Households optimally choose their productivity investment by maximizing equation (1) subject to (3) and (4), which yields the following policy rule (as a function of $q$ and $\bar{c}$):

$$z(q, \bar{c}) = \begin{cases} \frac{(q - \bar{c})(2\chi - 1)}{q} & \text{if } \chi > 0.5 \\ 0 & \text{if } \chi \leq 0.5 \end{cases}.$$  \hfill (5)

In what follows it will be assumed that $\chi > 0.5$ in order to ensure positive investment. Note that the optimal household investment can be rewritten in terms of the coefficient of relative risk aversion:

$$z(q, \bar{c}) = \frac{2\chi - 1}{r_{R}(q, \bar{c})}.$$  \hfill (6)

Not surprisingly, since innovation involves uncertainty an increase in the household’s relative risk aversion results in lower investment. The properties of $z(\cdot, \cdot)$ are summarized in proposition 1.

**Proposition 1.** The optimal decision (5) satisfies:

1. $z(q, 0) = 2\chi - 1$
2. $z_1(q, \bar{c}) > 0, z_{11}(q, \bar{c}) < 0$
3. $z(q, \bar{c}) < \lim_{q \to \infty} z(q, \bar{c}) = z(q, 0)$
4. $\frac{\partial z(q, \bar{c})}{\partial \chi} > 0$
The proofs of these results are straightforward and have been omitted. Proposition 1 follows directly from the properties of CRRA and DRRA. The most important feature of proposition 1 is that risk taking is lower when households face subsistence requirements \( z(q, 0) > z(q, \bar{c}) \). The remainder of this paper will be concerned with the causes and consequences of this wedge \( z(q, 0) - z(q, \bar{c}) \) that occurs as a result of subsistence requirements.

### 2.3 Disposable Income vs. DRRA

The introduction of subsistence requirements into the household's preferences introduces two constraints. First, as is standard in the literature, subsistence requirements impose a constraint on disposable income. Households must allocate \( \bar{c} \) of their income toward consumption and only the remaining \( y - \bar{c} \) of income can be allocated as they see fit (this will be referred to as the disposable income constraint). Second, as noted above, the introduction of subsistence requirements directly into the utility function results in decreasing relative risk aversion. As a result, the subsistence consumption requirement also acts as a constraint on risk-taking (referred to as the DRRA constraint). The distinction between the disposable income and the DRRA constraints is important because it will ultimately indicate the optimal policy. If the difference between innovation with and without subsistence requirements is largely due to the disposable income constraint this would imply that some sort of cash transfer to alleviate this constraint would be the optimal policy. On the other hand, if the DRRA constraint is the dominating force the appropriate response may be to create insurance markets to deal with this elevated risk aversion.

Unfortunately, the results in proposition 1 are not enough to distinguish between the disposable income and DRRA constraints. In order to separate the two constraints, it is necessary to construct a model where households face the same limitations on disposable income but are not subject to the increased levels of risk aversion that are a result of DRRA preferences. This new model uses an external subsistence requirement (not included in the preference specification) which imposes the disposable income constraint while leaving the household with the standard CRRA utility. The household's decision is given in equation (7).

\[
\begin{align*}
\max_{\hat{z}} & \quad \log(q(1 + \hat{z})) \chi + \log(q(1 - \hat{z}))(1 - \chi) \\
q(1 - \hat{z}) & \geq \bar{c} \\
q(1 + \hat{z}) & \geq \bar{c}
\end{align*}
\]

Note that only the first constraint is binding for the household. The optimal choice for the
While $\hat{z}$ is increasing in productivity for sufficiently low $q$ the optimal choice of investment converges to that of those not bound by subsistence for finite, rather than infinite, $q$. Based on equation (8) it is clear that $z(q, 0) \geq \hat{z}(q, \bar{c}) \geq z(q, \bar{c}) \quad \forall q$. Therefore the entire impact of subsistence consumption (the disposable income and DRRA constraints) can be quantified by:

$$\tilde{z}(q) \equiv z(q, 0) - z(q, \bar{c}) = 2\chi - 1 - (2\chi - 1) \frac{q - \bar{c}}{q} = (2\chi - 1) \frac{\bar{c}}{q}. \quad (9)$$

It is clear that $\tilde{z}'(q) < 0$, $\tilde{z}''(q) > 0$, $\lim_{q \to \infty} \tilde{z}(q) = 0$. Denote the share of $\tilde{z}(\cdot)$ that is due to the disposable income constraint as, $\phi^I(q)$, and the share that is due to the DRRA constraint as, $\phi^R(q)$, these expressions can be written explicitly as:

$$\phi^I(q) = \frac{z(q, 0) - \hat{z}(q, \bar{c})}{\tilde{z}(q)} \quad (10)$$

and

$$\phi^R(q) = \frac{\hat{z}(q, \bar{c}) - z(q, \bar{c})}{\tilde{z}(q)}. \quad (11)$$

It is straightforward to show that $\frac{\partial \phi^I(q)}{\partial q} < 0$ and $\frac{\partial \phi^R(q)}{\partial q} > 0$ and that there is a unique $\bar{q}$ such that $\phi^I(\bar{q}) = \phi^R(\bar{q})$, where $\bar{q} = (3 - 2\chi)\bar{c}/(4(1 - \chi))$. Figure 1 plots the optimal choice for both preference structures and the share of $\tilde{z}(q)$ explained by both constraints.

The figure below shows that when households are relatively poor, the disposable income constraint dominates their choices. Quite simply, deeply impoverished households are unable to afford the cost of innovation rather than being unwilling to risk it. However, as income (productivity) increases the DRRA constraint quickly dominates the disposable income constraint and becomes the sole driver of $\tilde{z}$ once $q > \bar{q}$. One question that may arise is how wealthy (productive) does a household have to be before the DRRA constraint dominates/becomes the sole driver of underinvestment. In order to answer this question, I will look at productivity relative to the subsistence requirement (i.e. $\bar{q}/\bar{c}$ and $\bar{q}/\tilde{c}$). The advantage of doing so is that these ratios only depend upon the probability of success, $\chi$.

Figure 2 sheds some light on the importance of disposable income versus the increased risk aversion suffered by households due to subsistence. Clearly the cut-off values are increasing
Figure 1: Disposable Income Effect vs DRRA effect
The figure on the left compares the optimal choice of investment for utility functions with both DRRA and CRRA for households who are bound by a subsistence consumption constraint and those who are not. The figure on the right looks at the relative importance of the disposable income and DRRA constraints.

and convex in the probability of success. This is rather intuitive, as certainty increases, risk aversion of any kind becomes less important; meaning that the disposable income constraint largely dominates the DRRA constraint. This has some interesting policy implications. Consider the case of unconditional cash transfers. If a program is interested in getting people to invest in low risk activities (buying a cow, acquiring more inventory, setting up a small store) then unconditional transfers will have the desired effect, because when probability of success is high the largest constraint is financial. If, on the other hand, the program is interested in getting people to do more risky things (adopt a high yield variety of crop, use an untested technology) simply plying them with cash will not relieve the most significant constraint households face: risk aversion. The implications for growth depend on whether growth enhancing activities are inherently risky. If long-run growth is achieved by taking risks (i.e. innovation rather than investing in cattle), unconditional cash transfers will not be enough to make a significant difference.

Now that the household’s choice has been sufficiently characterized, it is possible to discuss the growth rate of output. Proposition 2 presents the results that pertain to the aggregate growth rate.

Proposition 2. Under the model specified with equations (1)-(4), the aggregate growth rate of the
Figure 2: $\tilde{q}/\bar{c}$ and $\bar{q}/\bar{c}$ as a function of $\chi$.
This figure shows the productivity relative to subsistence consumption needed for households to be relatively more constrained and completely constrained by the DRRA effect for every possible probability of success.

economy is:

$$g_t(\bar{c}) \equiv \frac{Y_t}{Y_{t-1}} - 1 = (2\chi - 1)^2 \left(1 - \frac{\bar{c}}{Y_{t-1}}\right)$$

(12)

and satisfies the following properties:

1. $\frac{\partial g_t(\bar{c})}{\partial \chi} > 0, \quad \frac{\partial^2 g_t(\bar{c})}{\partial \chi^2} > 0$

2. $\frac{\partial g_t(\bar{c})}{\partial \bar{c}} < 0$

3. $\lim_{Y_{t-1} \to \infty} g_t(\bar{c}) = g_t(0) = (2\chi - 1)^2$

All proofs are included in the Appendix. While the results presented above are significant, they are by no means surprising. It has already been demonstrated that household investment was increasing in the probability of success and decreasing in subsistence consumption, which implies that the growth rate of output would be as well. Additionally, as household productivity increases, investment converges to the no subsistence benchmark, it would follow that the growth rate of the economy would exhibit the same pattern as output increases.

Unfortunately, given the kink in the household investment function the growth rate of output under the CRRA preferences with an external subsistence constraint depends on the distribu-
tion of household productivities and therefore not directly comparable to the one found in proposition 2. However, under two mild assumptions proposition 3 shows that output will be higher under CRRA utility.

**Proposition 3.** Assume that there are two economies, both with subsistence consumption constraints while one has DRRA preferences and the other had CRRA preferences such that the optimal choices for household investment are given by equations (5) and (8). In addition assume that at \( t-1 \) the distribution of household productivities and the success/failure draws are identical between the two economies. Then output under CRRA (\( \hat{Y}_t \)) is greater than output under DRRA (\( Y_t \)):

\[
\hat{Y}_t - Y_t = 2(1 - \chi)(2\chi - 1) \int_q^\theta (y - \bar{c})dG_{t-1}(y) + (2\chi - 1)^2\bar{c}(1 - G_{t-1}(q)) > 0
\]

Proposition 3 effectively says that if you take an economy at any point in time and assume that successfully increasing household productivity is not tied to the choice of utility function, then output will be higher under CRRA utility. This is consistent with what was shown in regards to individual household choices. Under DRRA, households invest (in some cases, significantly) less in innovation compared to CRRA. Because households on average succeed more than they fail (\( \chi > 0.5 \)) it makes sense that this under investment (\( \tilde{z} > z \)) would result in lower output.

At this point it is time to turn to a full theoretical model in order to understand the impact of subsistence consumption and decreasing relative risk aversion.

### 3 The Theoretical Model

#### 3.1 Production

The economy produces two goods, an agricultural good (\( Y_t^A \)) and a non-agricultural good (\( Y_t^{NA} \)). In addition, suppose the economy is populated with \( N \) households, who differ in their initial farm productivity. Following Matsuyama (1992) both goods are produced using weakly concave production functions, taking labor as their only input.\(^7\) Specifically, the non-agricultural

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\(^7\)The major contribution of Matsuyama (1992) was to highlight how structural change was different in an open economy as compared to a closed economy. He argued that countries would specialize in the production of goods for which they have comparative advantage. However, Restuccia et al (2008) showed that in 1985 the labor productivity in agriculture was 78 times higher in the top decile of countries than it is in the bottom decile. They also showed that non-agricultural labor productivity was only 5 times higher in the top decile than the bottom.
good is produced using an AK production function:

\[ Y_t^{NA} = B_t L_t \]  \hspace{1cm} (14)

where \( L_t \) is the labor force employed in the non-agricultural sector and \( B_t \) the productivity level which evolves according to the following learning-by-doing process:

\[ B_t = B_{t-1} (1 + H(L_{t-1})) \]  \hspace{1cm} (15)

where \( H(\cdot) \) is an increasing, weakly concave function of the non-agricultural labor force.\(^8\) Agricultural output is produced by Yeoman farmers, using household specific productivity (\( q \)) and labor supply (\( l \)).\(^9\)

\[ y_{it} = q_{it} l_{it}^\psi \]  \hspace{1cm} (16)

where \( 0 < \psi < 1 \). Total agricultural output is the sum total of production by the households:

\[ Y_t^A = \sum_{i=1}^{N} y_{it} \]  \hspace{1cm} (17)

Households are endowed with a unit of labor, the time not spent farming is allocated to working in the non-agricultural sector. Total employment in the non-agricultural sector is given by:

\[ L_t = \sum_{i=1}^{N} (1 - l_{it}) \]  \hspace{1cm} (18)

Letting \( A \) represent agricultural productivity and \( N \) represent non-agricultural productivity yields:

\[ \frac{A^T}{A^B} > \frac{N^T}{N^B} \Rightarrow \frac{A^T}{N^T} > \frac{A^B}{N^B}, \]

where \( T \) and \( B \) mean the top and bottom deciles, respectively. The above equation implies that the top decile has comparative advantage in agriculture, which means that a greater percentage of their labor force should work in agriculture. However, as shown in Restuccia et al (2008), only 5% of workers in the top decile work in agriculture compared to 82% in the bottom decile, which contradicts the result found in Matsuyama (1992). Given that economies are not specializing along the lines of comparative advantage, it stands to reason that there is something that is effectively closing these economies (or at the very least, parts of the these economies). Under these circumstances, it seems natural to consider the behavior of households in a closed economy setting.

\(^8\)The computational results will use the following functional form: \( H(L_t) = \phi \times (L_t / N) \).

\(^9\)Farmers employ only household labor in production, however they can sell their output in the market. Note that this implies that not all households will be farmers. Sufficiently low productivity households may opt into full-time non-agricultural work.
Aggregate production is the sum of the output in the agricultural and non-agricultural sector.

\[ Y_t = Y_t^A + p_t Y_t^{NA} \] (19)

where \( p_t \) is the relative price of non-agriculture in terms of agricultural goods. The equation for the relative price is given below.

### 3.2 Innovation

Households can stochastically increase their agricultural productivity by investing \( z \). Innovation is not free and requires the purchase of \( \rho z \) units of the non-agricultural good.\(^{10}\) Farmers successfully innovate with probability \( \chi \) and increase their productivity by the entirety of their investment \( z \). Failure to innovate occurs with probability \( 1 - \chi \) and as a result farmers lose \( \omega z \) of their output.\(^{11}\)

\[ q_{it} = \begin{cases} 
q_{it-1}(1 + z_{it}) & \text{w/ probability } \chi \\
q_{it-1}(1 - \omega z_{it}) & \text{w/ probability } 1 - \chi 
\end{cases} \] (20)

Intuitively this can be thought of in two ways. First, given the existence of informal risk sharing networks in developing countries it is unlikely that a single farmer would be liable for the entirety of his losses. Second, agriculture rarely fails in such a way that the farmer would lose all of the output associated with the new technology.\(^{12}\) It is also assumed that an unsuccessful innovation attempt only affects the current generation, the subsequent generation can produce using any of the previously discovered technologies (the optimal being the most productive). This means that between generation evolution of household technology is given by:

\[ q_{it} = \begin{cases} 
q_{it-1}(1 + z_{it}) & \text{w/ probability } \chi \\
q_{it-1} & \text{w/ probability } 1 - \chi 
\end{cases} \] (21)

This will ensure that household productivities do not fall to zero.\(^{13}\)

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\(^{10}\)This is similar to the assumption made by Donovan (2015) in regards to intermediate inputs. \( \rho \) will be used to calibrate the transition.

\(^{11}\)Note that this formulation implies that \( z \leq 1/\omega \) as it is impossible for a farmer to produce negative output.

\(^{12}\)\( \omega \) is calibrated so that the long run growth rate of output per agricultural worker is consistent with the data.

\(^{13}\)This would have devastating consequences, over time an increasing share of households have zero productivity which means that there are wild swings in agricultural and non-agricultural output as the labor force shifts to compensate for good and bad periods.
3.3 Preferences

Households are active for one period and have preferences over both the agricultural and non-agricultural good. They must consume enough of the agricultural good to satisfy their subsistence constraint and only receive utility from consumption net of subsistence. The utility specification is given below.

\[ U_{it} = \alpha \log(c_{it}^A - \bar{c}) + (1 - \alpha) \log(c_{it}^{NA}) \]  
(22)

Households earn income from farm labor and working for a wage in the non-agricultural sector.\(^\text{14}\) This income is divided between consumption of the agricultural and non-agricultural goods as well as spending on productivity investment.

\[ c_{it}^A + p_t c_{it}^{NA} + p_t \rho z_{it} = y_{it} + (1 - l_{it}) w_t \]  
(23)

where \( p_t \) is the relative price of the non-agricultural good in terms of agriculture. Equation (14) implies that the wage is \( w_t = p_t B_t \). Maximizing (22) subject to (23), yields the optimal choices for the consumption of the agriculture and non-agricultural goods:

\[ c_{it}^A = \bar{c} + \alpha (y_{it} + (1 - l_{it}) w_t - p_t \rho z_{it}) \]  
(24)

\[ c_{it}^{NA} = \frac{(1 - \alpha)(y_{it} + (1 - l_{it}) w_t - p_t \rho z_{it}) - \bar{c}}{p_t} \]  
(25)

Equations (24) and (25) imply that the indirect utility function is:

\[ V_{it} = \eta_t + \log(y_{it} + (1 - l_{it}) w_t - p_t \rho z_{it}) \]  
(26)

where \( \eta_t \equiv (1 - \alpha) \log(1 - \alpha) + \alpha \log(\alpha) - (1 - \alpha) \log(p_t) \). As in the previous section, equations (24)-(26) show that the subsistence consumption requirement imposes two constraints on households. First, subsistence consumption limits the household’s disposable income. Households must spend \( \bar{c} \) of their income on agricultural consumption, therefore they only have discretion over the remaining \( y_{it} + (1 - l_{it}) w_t - \bar{c} \). This remaining income must be split between additional agricultural consumption, non-agricultural consumption, and investment \((z)\). Second, allowing for \( c_{it} = y_{it} + (1 - l_{it}) w_t - p_t \rho z_{it} \), note that the indirect utility function exhibits decreasing relative risk aversion. Like in the previous section, the analysis below is concerned not only

\(^{14}\)Employment in the non-agricultural sector is without risk. This can be thought of as households being employed in multiple occupations in order to insure against risk, as described in Banerjee and Duflo (2007). The inclusion of exogenous risks like job loss would only strengthen the results presented later in the paper as it would make households less likely to take on the risk of innovation.
with the effect that subsistence consumption has on aggregate outcomes like growth, but also the roles that disposable income and DRRA constraints play.

Aggregating over (25) and setting it equal to (14) shows that the relative price is:

\[ p_t = \frac{(1 - \alpha)(Y_t^A - \bar{c}N)}{\alpha(Y_t^{NA} - \rho Z_t)} \]

(27)

where \( Z_t = \sum_{i=1}^{N} z_{it} \). The household must decide how much labor to supply to farming and the amount to invest in innovation. The labor supply decision must occur prior to resolving any uncertainty. Similar to Matsuyama (1992) this model abstracts away from savings in an effort to focus on the primary mechanism of the model, risk taking under subsistence constraints. While savings is often focused on as a primary mechanism for poverty alleviation, this model is concerned with the actions of the extremely poor leading to two main reasons for the abstraction. First, as documented in the literature (see Banerjee and Duflo, 2011 and Karlan and Appel, 2011 for excellent overviews), the poor have relatively low financial savings which means that removing it from the model should have very little effect on the household’s decision. Second, among the poor, intergenerational investments in children tend to be health or education related (thus increasing the child’s productivity), rather than financial. This type of intergenerational transfer is already captured through the household’s investment in agricultural productivity.\(^{15}\)

\(^{15}\)The model presented in this section can be extended to include savings by giving households warm glow preferences over bequests, so that utility function is: \( u_{it} = \alpha \log(c_{it}^A - \bar{c}) + (1 - \alpha) \log(c_{it}^M) + \beta \log(a_{it+1}) \). In doing so, the household’s bequests, \( a_{it+1} \) become of functions of their disposable income:

\[ a_{it+1} = \left( \frac{\beta}{1 + \beta} \right) \left( \frac{y_{it} - \bar{c}}{p_t} \right) \]

where \( y_{it} = q_{it}^A \psi_{it} + (1 - l_{it})w_t + R_t a_{it} - p_{it}\rho z_{it} \) is the household’s income. In this set-up bequests act as a source of income and as a source of heterogeneity (richer households have richer offspring due to greater bequests). As seen in the previous section, richer households invest more in increasing their productivity, therefore savings offers an additional route by which households can improve their productivity. However, both of the effects of savings are already captured by the heterogeneity present in farm productivity. Higher productivity \( (q) \) leads to higher bequests and greater investment in innovation.

There is conceivably one role in which savings plays a different role than farmer productivity; it provides the farmer with a source of certain income. In the early stages of development, poor farmers who are undertaking risky innovation may wish to diversify their activities by shifting some of their labor into more certain jobs (non-agricultural production). By shifting labor away from agriculture in an effort to find more certainty, farmers will face a lower return on their investment and therefore invest less. Savings can provide the certainty that the farmer desires without requiring them to lower their returns by changing their labor supply, which could induce them to invest more in innovation. However, this is most likely a minor concern. As seen in the equation above, the poor farmers who would benefit from these buffer savings are also the least likely to have them as the poor save at a disproportionately lower rate. Additionally, empirical evidence suggests that these buffer stocks do not exist in reality and the poor tend to take out loans or sell off productive assets in the event of a negative shock. Again for excellent discussions of the savings behavior of the poor, see Banerjee and Duflo (2011) and Karlan and Appel (2011).
The household’s problem is:

\[
\max_{z_{it}, l_{it}} \log(q_{it-1}(1+z_{it}))^{\psi_{it}} + (1-l_{it})w_{it} - p_{it}\rho z_{it} - \bar{c} \chi + \log(q_{it-1}(1-\omega z_{it}))^{\psi_{it}} + (1-l_{it})w_{it} - p_{it}\rho z_{it} - \bar{c}(1-\chi)
\]

(28)

The resulting first order condition for household innovation is:

\[
z_{it} = \frac{q_{it-1}l_{it}^{\psi_{it}} + (1-l_{it})w_{it} - \bar{c}}{q_{it-1}l_{it}^{\psi_{it}} - p_{it}\rho} \left( q_{it-1}l_{it}^{\psi_{it}}(\chi - (1-\chi)\omega) - p_{it}\rho \right) \left( \omega q_{it-1}l_{it}^{\psi_{it}} + p_{it}\rho \right).
\]

(29)

Unfortunately, \( l_{it} \) does not have a closed form solution, however, the first order condition is given below.

\[
-\frac{\psi(1-l_{it})w_{it} + l_{it}w_{it} - \psi \bar{c}}{q_{it-1}l_{it}^{\psi_{it}} + (1-l_{it})w_{it} - \psi \bar{c}} + \frac{\rho \chi p_{it}}{q_{it-1}l_{it}^{\psi_{it}} + \rho p_{it}} - \frac{\rho(1-\chi)\psi p_{it}}{q_{it-1}l_{it}^{\psi_{it}} - \rho p_{it}} + \psi = 0
\]

(30)

It is possible to examine the comparative statics for the household’s choice variables by using the parameters chosen for the calibration. In addition to the calibrated parameters the two first order conditions also rely on values for the relative price and the wage. These values are determined using the first four periods of the simulations in the next section, therefore each of the lines corresponds to a different time period. The comparative statics are done by graphing the household’s choice variable across a range of farmer productivities.

Figure 3: Comparative Statics: Labor Supply and Investment
The figure on the left looks at how labor supplied to farming varies across productivity, while the figure on the right looks at the relationship between investment in innovation and productivity. Each line corresponds to a different time period of the simulations found in the next section (black/solid–period 1, red/dashed–period 2, blue/dotted–period 3, green/dashed-dot–period 4. Note that both the wage and the relative price are increasing as the simulation progresses.
Not surprisingly, both labor supplied to agriculture and investment are increasing in farm productivity. This is because an increase in farm productivity results in greater returns to farming and innovation. Also, note that as the wage increases over time it simultaneously causes farmers to reduce their labor supply to agriculture while increasing their innovation. The labor supply effect is due to farmers substituting toward higher wages, while the investment effect is the combination of greater income due to higher wages and the fact that the risk of innovating has fallen because farmers are devoting less time to agriculture.

4 Quantitative Results

4.1 Parameters

There are eleven parameters that need to be calibrated, unfortunately only one of which has a direct empirical counterpart: the labor share in agriculture. Following Donovan (2015), $\psi$ is set to be 0.4. Of the remaining parameters, four ($q_{\text{min}}, \beta, B_0, \bar{c}$) deal with the initialization of the model, three determine the long-run outcomes ($\alpha, \phi, \omega$), and $\rho$ determines the length of the transition. The long-run in this model would occur when the share of employment in agriculture stabilizes. Restuccia et al. (2010) show that among the countries in the top income decile, the share of employment in agriculture post-1985 was low (< 5%) and stable, for the purposes of this paper it will be assumed that these countries have reached their long-run equilibrium. In the model presented in the previous section, a stable employment share would also imply stable growth rates of output per agricultural and manufacturing worker in the long-run because growth in these industries would solely by driven by household investment and the learning-by-doing externality. This would mean that the long-run growth rates of the model are entirely determined by $\omega$ and $\phi$ for a given $\chi$. These parameters are calibrated using the growth rates from the countries in the top income decile from the Restuccia et al. (2010) dataset. Note that this does not imply that the growth rates are exogenously determined during the transition to the long-run as they still depend on how households respond to the disposable income and DRRA constraints. The World Development Indictors are used to pin down the long-run employment in agriculture in the United States, which determines the value of $\alpha$. In order to initialize the model, I use data from Restuccia et al. (2008). The initial productivity distribution is assumed to be Pareto with minimum value $q_{\text{min}}$ and shape parameter $\beta$. In 1985, 82% of all employment in the countries located in the bottom income decile was in the agricultural sector. This value is used to determine the spread of initial farm productivity draws ($\beta$). The minimum farm productivity is chosen so that the lowest productivity household could satisfy
their subsistence requirement by themselves \( (q_{min} = \bar{c} + \epsilon, \text{ for } \epsilon \text{ small}) \). Additionally, Restuccia et al. (2008) showed that non-agricultural production was 29.73 times more productive than agricultural production \( (B_0) \) in these countries.\(^{16}\) The subsistence parameter is normalized to 1.\(^{17}\) The cost of innovation \( \rho \) is set so that the transition from the initial period to the long-run (low/stable employment in agriculture) is eight generations, which is consistent with the US experience (Acemoglu, 2008).\(^{18}\) The specific values for the parameters can be found in table 1.\(^{19}\)

### 4.2 Baseline Results

This section of the paper looks in depth at the roles that risk and subsistence consumption have not only on household behavior but the macroeconomy as a whole. In particular, the question of interest is how do the disposable income and DRRA constraints affect household investment, economic growth, and structural change? The model presented in the previous section will be used as the baseline. In order to determine the effects of the disposable income and DRRA constraints, variations on the baseline model will be used. These variations will include changing the probability of successful innovation from \( \chi = 0.75 \) to \( \chi = 1 \), removing the subsistence requirement, and changing household preferences from DRRA to CRRA (this is done in the same way as in section 2, where household’s face external subsistence constraints). In total there are six different preference/risk/subsistence variations of the baseline model. In order to make

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\(^{16}\) The initial productivity in the non-agricultural sector differs across simulations. It is calibrated so that the ratio of initial productivity per worker in the non-agricultural and agricultural sectors is consistent with the data.

\(^{17}\) The percentage of average consumption that corresponds to subsistence varies across the literature. Steger (2000) shows using poverty lines that for low income countries subsistence requirements correspond to between 85-95% of average income. In the initial period of the simulations below, the ratio of subsistence consumption to per capita income falls within this range. Chatterjee and Ravikumar (1999) show that the subsistence constraint corresponds to 58-80% of average consumption. However, these estimates are from India from 1974-1985 which had an employment share in agriculture of 33% and was in the third lowest decile for income. Therefore, these estimates would not correspond to the initial period of the model. That being said, the model does predict that a subsistence-average consumption ratio that is roughly consistent with the upper range of the data for those economies with employment shares near 33%.

\(^{18}\) It is likely that the \( \rho \) (the cost of transforming non-agricultural output into the investment good) is not constant and depends upon the state of the economy, though it is not immediately clear whether \( \rho \) should be increasing or decreasing over time. Also, while the length of the transition is pinned down by the choice of \( \rho \), the transition itself is still endogenous as it depends upon the investment choices of the households. The choice of \( \rho \) allows the model to match the US experience in an attempt to make reasonable statements about the impact of subsistence consumption and risk taking on household choices and macroeconomic outcomes. It should be noted that while the model is not initialized using U.S. data, it can reasonably said that the current bottom decile of countries provides a reasonable approximation. When the US had an agricultural share of 0.82, its per capita income was $1296 (Maddison Project, 1990 Dollars) marginally different from the per capital income of $1239 in the bottom decile countries.

\(^{19}\) The calibration results can be found in the Appendix.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.1</td>
<td>Preference Parameter: Utility from Agricultural Consumption</td>
<td>World Databank–World Development Indicators.</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.4</td>
<td>Labor Share in Agriculture Production</td>
<td>Donovan (2015)</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>1</td>
<td>Subsistence Consumption</td>
<td>Normalized</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.605602</td>
<td>Long Run Growth Rate in the Manufacturing Sector</td>
<td>Restuccia et al (2010)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.75</td>
<td>Probability of Successfully innovating</td>
<td>Free</td>
</tr>
<tr>
<td>$\rho$</td>
<td>43</td>
<td>Cost of Innovation</td>
<td>Acemoglu (2008)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.4</td>
<td>Agriculture loss due to failure to innovate</td>
<td>Restuccia et al (2010)</td>
</tr>
<tr>
<td>$B_0$</td>
<td>Varies</td>
<td>Initial Manufacturing Productivity</td>
<td>Restuccia et al (2008)</td>
</tr>
<tr>
<td>$N$</td>
<td>1000</td>
<td>Population Size</td>
<td>Scale</td>
</tr>
<tr>
<td>$q_{min}$</td>
<td>1.0001</td>
<td>Distribution Parameter</td>
<td>$q_{min} = \bar{c} + \epsilon$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>14.2</td>
<td>Distribution Parameter</td>
<td>Agriculture is 82% of the labor force in period 1. Restuccia et al (2008)</td>
</tr>
</tbody>
</table>

Table 1: Parameter Values

Before moving forward it is important to discuss how these different specifications will show the impacts of the disposable income and DRRA constraints. The first thing to note is that for two households that have the same farm productivity and the same non-agriculture wage, their disposable income could differ if they have different preferences. If the household has CRRA preferences, their disposable income is simply their total income net of subsistence, in other words this household would be willing to consume the subsistence amount. However, a household with DRRA preferences would never consume the subsistence amount because doing so would result in infinite marginal utility, therefore they have a smaller amount of disposable income (this will be referred to as the marginal utility constraint). This difference in disposable incomes can be determined by looking at the differences between the $CRRA/\chi = 1/\bar{c} > 0$ and $DRRA/\chi = 1/\bar{c} > 0$ models. Because household in neither model face uncertainty, the difference between the two is completely driven by the difference in disposable income.

Determining the effects of the increased risk aversion due to DRRA is not as simple as comparing the $CRRA/\chi = 0.75/\bar{c} > 0$ and $DRRA/\chi = 0.75/\bar{c} > 0$ models, because, as mentioned above, households in these model have different levels of disposable income. However, by comparing the differences between the models with certainty and uncertainty across preferences, it is

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20 This implies that the model in which household's have DRRA preferences, face uncertainty, and are bound by subsistence requirements is denoted as: $DRRA/\chi = 0.75/\bar{c} > 0$. 

---
possible to see how different risk profiles effect both macroeconomic and household outcomes. Finally, by varying the level of subsistence between 0 and 1 it is possible to determine the full effect of both the disposable income and DRRA constraints. However, most of this analysis will occur at the household level because removing subsistence from the model drastically alters the structure of the economy and it is difficult to make worthwhile comparisons.

The remainder of this section will be broken up into two parts. The first part will look at the macroeconomic effects of the different model specifications, while the second investigates the household level consequences.

### 4.2.1 Macroeconomic Outcomes

This paper is concerned with two macroeconomic outcomes: the rate of structural change by which the economy transforms from agrarian to non-agrarian focused production and the growth of the economy as a whole. Starting with the structural change of the economy, the measure used will be the share of employment in the agricultural sector. This is plotted for each of the six different simulations in figure 2 below.\(^{21}\)

![Figure 4: The share of workers employed in agriculture over time](image)

This figure plots the average share of workers employed in agriculture for each of the six different model specifications over time.

Subsistence consumption plays a rather large role in the share of workers employed in agriculture. In the models without subsistence requirements, there is no lower bound on agricultural production and therefore these economies need fewer workers to satisfy the agricultural demands of the consumers. Before discussing the impact that different preference structures have under subsistence requirements, it is necessary to discuss the long-run equilibria of the

\(^{21}\)The results presented in this figures and the ones below it are the averages across the 20 different productivity draws. The draws for initial productivities and success/failure are the same across the six different sets of simulations.
models. Figure 4 shows that the models that have subsistence requirements converge to different long-run employment shares depending on whether or not households face uncertainty. This is a result of the evolution of household productivities. Under certainty, any investment necessarily results in an increase in household productivity. In contrast, in an economy with uncertainty there will be a fraction of households that do not realize productivity increases despite investments. This means that as the productivity of the non-agricultural sector increases more households will find it suboptimal to continue as farmers in an economy with uncertainty because their own productivity does not keep pace.

Looking at the structural change in figure 4c a reasonable question to ask is: how well does this model match the available data? Empirically testing models of structural change tends to be difficult due to the lack of historical employment share data across a wide range of countries. Following Duarte and Restuccia (2010), the model presented in the previous section is empirically tested using 50 years of employment data.\textsuperscript{22} The data is from the WDI and the International Labour Office database and it includes 27 countries covering the years 1960-2010. The countries in the dataset appear at different stages of development and therefore the first empirical observation for each country may not correspond to the initial condition for the simulated model. In order to correct for this each country's starting point is shifted so that the first empirical observation is the same as the simulated value.\textsuperscript{23} The results of this empirical test are presented in figure 5 where the countries are aggregated by region (Asia, Africa, and Latin and South America). Figures showing the empirical test by country can be found in the appendix.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Empirical Test of the Rate of Structural Change Across Countries}
\end{figure}

This figure compares the model generated rate of structural change (black line) with the rates of structural change in countries across three regions (Asia, Africa, and Latin and South America).

\textsuperscript{22}This allows me three data points for each country, in contrast in Duarte and Restuccia (2010) the length of a period is a year which means they have 50 observations for each country.

\textsuperscript{23}In this set up a country with an observed labor share in agriculture of 60% would start in generation 4. In the case that a country's initial labor share is greater than what is observed in the simulation, I assume the country starts in generation 1.
Figure 5 shows that the model fits the data quite well in Latin and South America as well as Africa. However, the Asian countries do not seem to fit the pattern set forth by the model, although the simulations do predict the observed employment shares for India, the Philippines, and Sri Lanka. What explains the variation in the model’s explanatory power? A reasonable explanation is that the economic experience of countries in Asia was extremely different than those in Africa and the Americas over this time period. In particular, Asian countries experienced a large increase in international trade which, as Matasuyama (1992) predicted, resulted in different outcomes in terms of structural change.24

Overall this empirical test shows the importance of the open economy assumption. The significantly more open Asian countries displayed a much faster rate of structural change. However, as discussed in section 3 and shown in figure 5, the open economy assumption may not be completely relevant for all countries and in fact a closed economy assumption could better reflect some countries experiences. The empirical test shows that this model is policy relevant for a wide range of countries, albeit with the caveat that it may not fully represent the realities faced by those countries whose rapid growth is fueled by export promotion. At this point the discussion will turn to explaining the impacts of the disposable income and DRRA constraints.

It should be noted that as the discussion of the macroeconomic effects of subsistence consumption and risk taking progresses, it is not possible to quantify the entire disposable income effect because the economies that do not have subsistence requirements are structurally different from those that do. Therefore the focus will be on discussing the marginal utility constraint (the reduced disposable income as a result of having DRRA preferences) and the risk aversion constraint. Given that the household preference for agriculture is relatively low, the main determinants for the share of workers employed in agriculture is the subsistence requirement and the average farm productivity. Farm productivity is determined by the cumulative effect of innovation and as it increases fewer workers are needed in the agricultural sector. The goal is to determine the impact that the increased risk aversion and reduced disposable income (marginal utility constraint) that result from DRRA preferences have on the rate of structural change. In order to determine this impact a counterfactual is constructed to show how households who are bound by the disposable income constraints of DRRA preferences would respond to risk if they had CRRA preferences. Starting with figure 4b, the difference between the model with risk and the one without quantifies the effect CRRA preferences on the time path of structural

24The appendix includes figures showing how trade as a percent of GDP changed over the same period in 25 of 27 countries (Tanzania and Zambia did not have sufficient data). Asian countries saw significant increases (Turkey’s trade as a percentage of GDP increased nearly 10 fold) and while countries in other regions saw increases they were not nearly as drastic as those in Asia (Mexico had the highest increase of a non-Asian country at three fold).
change. Define this difference as the vector $\Omega$:

$$
\Omega = \left\{ (CRRA/\chi = 0.75/\ddot{c} > 0)_t - (CRRA/\chi = 1/\ddot{c} > 0)_t \right\}_{t=1}^T
$$

$\Omega$ then quantifies how, for a given level of disposable income, a CRRA household would respond to the introduction of risk. Next, $\Omega$ is added to the vector $\{(DRRA/\chi = 1/\ddot{c} > 0)_t\}_{t=1}^T$. This new vector of employment shares then shows how households who have the disposable income constraints that occur with DRRA preferences would respond to risk if they had CRRA preferences. Figure 6 shows the result.

![Figure 6](image)

**Figure 6: Labor share employed in agriculture for DRRA ($\chi = 0.75/\ddot{c} > 0$), CRRA ($\chi = 0.75/\ddot{c} > 0$), and counterfactual data**

This figure plots a counterfactual time-series along with the baseline model time-series for the DRRA ($\chi = 0.75/\ddot{c} > 0$) and CRRA ($\chi = 0.75/\ddot{c} > 0$) simulations in order to quantify the relative importance of the constraints on disposable income and increased risk aversion over time.

In figure 6 there are two differences of interest. The first is the difference between the counterfactual data and the CRRA $\chi = 0.75/\ddot{c} > 0$ model which quantifies the disposable income constraint that results from the Inada condition on marginal utility and the second is the difference between the counterfactual data and the DRRA $\chi = 0.75/\ddot{c} > 0$ which quantifies the impact of the increase risk aversion that results from DRRA preferences. Based on the results in figure 6 in the early stages of development when households are still poor it is the marginal utility constraint that determines most of the difference in employment share in agriculture, however as economies grow the DRRA constraint begins to dominate. This is consistent with the intuition that was built with the simple model, when households are poor investment (and thus farm productivity) is lower because of an inability to afford innovation. However, as incomes grow constraints on disposable income become less pertinent and the household’s risk preferences take over.

Moving away from the question of structural change, it is important to understand how DRRA
preferences effect the growth rates of agriculture, non-agriculture, and output. Because the issue of structural change is usually thought of as one of agricultural productivity, it seems reasonable to begin this discussion by looking at the growth rate of output per agricultural worker over time, which are shown in figure 7.

Figure 7 shows that the economies that have both risk and DRRA preferences have significantly lower growth rates during the early stages of development. Based on the discussion regarding the share of employment in agriculture and the simple model presented in section 2, it can be said that this is clearly a combination of financial constraints and increased levels of risk aversion. In order to quantify the impacts of the reduced disposable income and increased risk aversion that result from DRRA preferences, it is possible to do a counterfactual similar to the one above. Once again, the difference between the models with and without risk for those households that have CRRA preferences and subsistence requirements is calculated. This difference is then subtracted off of the time-series from the $DRA / \chi = 1 / \bar{c} > 0$ model, which yields a counterfactual whose households have the disposable income of households with DRRA preferences, but the risk profile of those with CRRA preferences. This counterfactual is plotted in figure 8.

Consistent with the evidence thus far, the reduced disposable income (marginal utility constraint) plays a significant role in the early stages of development when households are poor. Yet, once again, as the economy develops the increased risk aversion starts to drive the aggregate outcomes. The final aggregate outcome to look at is the growth rates non-agricultural output and output, these results are shown in figure 9.\footnote{The real GDP in figure 9 is calculated using a two period chain weighted method. Because this model deals with structural change, selecting a single base year tends to over/under state growth at the extremes. The chain-weighted method avoids these issues.}
Figure 8: Growth rate of output per agricultural worker for DRRA ($\chi = 0.75/\bar{c} > 0$), CRRA($\chi = 0.75/\bar{c} > 0$), and counterfactual data. This figure plots a counterfactual time-series along with the baseline model time-series for the DRRA ($\chi = 0.75/\bar{c} > 0$) and CRRA($\chi = 0.75/\bar{c} > 0$) simulations in order to quantify the relative importance of the constraints on disposable income and increased risk aversion over time.

Figure 9 shows that decreasing relative risk aversion has a significant impact on both the growth rates of non-agricultural and aggregate output. During the first eight generations of development the CRRA ($\chi = 0.75/\bar{c} > 0$) and DRRA ($\chi = 0.75/\bar{c} > 0$) economies grow at an average annual rate of 1.16% and 1.12%, respectively. Over the course of a generation, if the two economies have the same initial income the one with CRRA preferences would be 13.3% richer than the economy with DRRA preferences. This a rather large impact for DRRA preferences to have on aggregate outcomes. At this point it is time to examine household choices in order to motivate any policy recommendations that could alleviate the aggregate effects of subsistence consumption and DRRA preferences observed in this section.

4.2.2 Household Investment

The aggregate results indicate that the disposable income and DRRA constraints have significant impacts on household investment. However, because the lack of the subsistence constraint fundamentally alters the structure of the economy, it is not possible to understand the full extent of the disposable income constraint. Therefore it is necessary to look at the household level behavior.

Before looking at the results, it is important to make a distinction between “farmers” and “non-farmers”. For the purpose of this paper a farmer is someone who supplies at least 25% of his labor endowment to farming activities. The reason why this distinction is important is that...
those individuals who supply little or no labor to farming also do not invest in innovation. If the average level of innovation was calculated across all households in the society, those economies with small shares of employment in agriculture would appear to have extremely low rates of investment. Setting the cut-off at 25% of the labor supply ensures that the households that are being examined have the proper incentive to invest.

Three different model specifications will be considered: $CRRA/\chi = 0.75/\bar{c} = 0$, $CRRA/\chi = 0.75/\bar{c} > 0$, and $DRRA/\chi = 0.75/\bar{c} > 0$. Note that all of the simulations include risk, which implies that households in the first two simulations with CRRA preferences have the same risk profiles and therefore differences in their investment will be due to the disposable income constraint. By comparing the difference between the DRRA and CRRA simulations (both with subsistence requirements) it is possible to determine the impact of the DRRA constraint. Figure 6 plots the average household investment over time across these three simulations as well as the decomposition of the disposable income ($\phi^I$) and DRRA constraints ($\phi^R$).

During the transition, households without subsistence requirements invest the most as they are the ones the face the fewest constraints. Households in the $CRRA/\chi = 0.75/\bar{c} > 1$ simulation see a reduction of investment compared to the no subsistence model because of the constraints on their disposable income. Finally, the households in the $DRRA/\chi = 0.75/\bar{c} > 1$ invest the least because they not only have constraints on their disposable income, but also experience increased levels of risk aversion. In the long-run the average investment tends to equalize across the three simulations as the disposable income and DRRA constraints reduce. Decomposing the two constraints, figure 10b shows that for the first six generations of development the con-

Figure 9: Growth rates of non-agricultural and aggregate output. This figure looks at the growth rates of aggregate output and non-agricultural production during the transition to the long-run equilibrium for employment.
constraint on disposable income is the most pressing concern to households. On average, by the sixth generation, the subsistence requirement corresponds to 68% of the median household’s consumption and 55% of the mean’s. This is in line with the results for subsistence consumption from Chatterjee and Ravikumar (1999) for 1980’s India and suggests that these economies are still very much “developing” by the time that the DRRA constraint becomes the household’s primary concern.

One of the difficulties with attempting to decompose the effects of the disposable income and DRRA constraints by looking at economy wide averages over time is that in any given generation, the household’s state is determined by a combination of the two constraints which have influenced not only the aggregate variables \( (p_t, w_t) \), but also the household level variables \( (q_t) \). This makes it difficult to isolate the true impact that both of these constraints have for a given state of the world. To solve this problem, I take the states of the economy that are pertinent to the household \( (p_t, w_t, q_{it-1}) \) from the \( DRRA/\chi = 0.75/\hat{\epsilon} > 0 \) model and then simulate their choices using CRRA preferences with and without the subsistence requirement. This means that for a given farm productivity \( q_{it-1} \), households with different preferences and subsistence requirements will still face the same trade-offs in terms of investment and earnings from non-agricultural labor. Figure 11 plots the results of this exercise.

Consistent with the past set of results, households without subsistence constraints invest significantly more than those that have them when farm productivity is low. Low productivity
Figure 11: Investment by farm productivity.
Figure (a) plots investment across household productivities for different risk profiles and preferences. In each simulation households face risk. Figure (b) decomposes the effects of the disposable income and DRRA constraints across farm productivity.

households tend to be limited in their resources, which causes them to invest less in innovation. However, as productivity increases constraints on disposable income become less important and households tend to make their investment decisions based on risk preferences. Given the analysis thus far, it is clear that in the early stages of development the elimination of financial constraints will provide the greatest gain to households with DRRA preferences.

5 Policy Analysis: Cash Transfers

The next step is to consider policy that would help mitigate the effects of the disposable income and risk aversion constraints. The results above show that in the early states of development, the disposable income constraint is the most pressing concern for poor households. This would imply that the optimal policy would be one that would increase the household's income (food security), rather than lower its risk. An interesting debate in development economics recently has been the relative efficiency of conditional versus unconditional cash transfers. This section will focus on examining the relative impacts of each type of transfer as well as determining the optimal form of the transfer for different circumstances. Before looking at the results from the general model, analytical results will be derived using the model in section 2 in order to build intuition.
5.1 A Simple Model of Cash Transfers

Preferences (DRRA) and the evolution of household productivities remain the same as in section 2, only now the budget constraint includes a cash transfer:

\[ c_t = y_t + CT_t \]  

(31)

where \( CT_t \) denotes the cash transfer at time \( t \) (the cash transfers are financed by aid). There are two possible types of cash transfers: conditional and unconditional. The unconditional transfer is given regardless of the amount of innovation, so that \( CT_t = \xi \), while the conditional cash transfer is determined by the size of the investment: \( CT_t = \zeta z_t \). Under these assumptions, the optimal choice of investment for the conditional and unconditional cash transfers are:

\[
\begin{align*}
   z^C(q, \bar{c}, \zeta, \chi) &= (q - \bar{c})(2\chi - 1)\zeta \frac{q^2 - \bar{c}^2}{q^2} \\
   z^U(q, \bar{c}, \xi, \chi) &= (q - \bar{c} + \xi)(2\chi - 1) \frac{q}{q}
\end{align*}
\]  

(32)

where \( z^C \) and \( z^U \) denote investment under conditional and unconditional cash transfers, respectively. I assume that \( \zeta < q_{min} \) to ensure that investment under conditional cash transfers is positive. To simplify the exposition, the analytical results relating to household investment under each type of cash transfer have been placed in the appendix. Figure 12 plots the two household investment curves as a function of the probability of success.

![Figure 12: Conditional versus unconditional cash transfers.](image)

This figure shows that when the probability of success is low, households need extra incentive to invest in innovation, which is accomplished through the conditional cash transfer. However,
as the probability of success increases risk constraints become less important and the household becomes more concerned with financial constraints. It should be noted that the analytical results in the appendix show that the crossing point in figure 12 only occurs when the financial gain from the unconditional cash transfer is larger than the conditional. When the financial gains from each transfers are the same, the conditional cash transfer outperforms the unconditional regardless of the probability of success.

5.2 Full Model with Cash Transfers

As in the previous subsection with the simple model, households’ budget constraints are now augmented by a cash transfer. The conditional cash transfer comes in the form of a subsidy on the price of innovation. For every unit of innovation the household purchases, they are given $\zeta p_t \rho$ units of the agricultural good, so that their budget constraint is given by:

$$c_i^A + p_t c_i^{NA} + (1 - \zeta) p_t \rho z_{it} = y_{it} + (1 - l_{it}) w_t. \quad (33)$$

In the simulations that follow $\zeta$ will be set at 0.25. The unconditional cash transfer, $\xi_t$, will be provided to the household regardless of investment, so that the budget constraint is:

$$c_i^A + p_t c_i^{NA} + p_t \rho z_{it} = y_{it} + (1 - l_{it}) w_t + \xi_t. \quad (34)$$

Two values for $\xi_t$ will be considered in the following simulations. First, I consider a constant value for $\xi$ equal to 5% of the subsistence level. Second, $\xi_t$ is constrained to be equal to the average cash transfer for each period under the conditional cash transfer ($\bar{z}_t^C$). Therefore,

$$\xi_t = p_t \ast \zeta \ast \bar{z}_t^C \quad (35)$$

Much like in figure 12, the relative effectiveness of each type of cash transfer will be determined across a wide range of probabilities of success. The full model is simulated using each of the three types of cash transfer (conditional, constant unconditional, variable unconditional) across probabilities of success ranging from 0.55 to 0.85. Because the probability of success differs greatly between each simulation, it is difficult to determine the relative impact that each type of cash transfer has on growth, however, it is possible to determine their effect on individ-

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27 There are several reasons why an unconditional cash transfer would provide a larger financial gain than a conditional one. The most intuitive reason would be that it requires lower administrative costs to implement an unconditional cash transfer because the aid agency would not have to pay someone to observe the actions of the households.
ual household investment. As in figure 10 above, only those households that supply more than 25% of their time endowment to farming will be considered farmers. Figure 13 shows the ratio of household investment for those households with conditional cash transfers and those with unconditional \((z^C / z^U)\).

Figure 13: Conditional versus Unconditional Cash Transfers
Figure (a) shows the ratio of household investment in innovation between conditional and constant unconditional cash transfers. Figure (b) shows the ratio of household investment in innovation between conditional and variable unconditional cash transfers.

In figure 13 it is clear that for those households with the lowest probability of success, conditional cash transfer are significantly better at increasing innovation. This is consistent with the intuition developed using the simple model, unconditional cash transfers perform better when the investments have a higher probability of success because households are only facing financial constraints rather than increased risk aversion.\(^{28}\) One of the more interesting results presented in figure 13 is that despite the fact that \(\bar{\xi}\) represents a significantly larger financial transfer than \(\xi_t\), it is relatively less effective at increasing household innovation. In order to explain this outcome, figure 14 looks at the share of employment and the growth rate of output per worker in agriculture for the model where the probability of success is \(\chi = 0.60\) across each type of cash transfer.

Looking first at the share of employment in agriculture, it is clear that the constant unconditional cash transfer significantly lowers the number of farmers (even when compared to the conditional cash transfer). This reduction in the share of labor employed in the agricultural sector is due to the fact that food aid effectively makes each farmer more productive by allowing

\(^{28}\)The “humps” at the end of the simulation in figure 13b are a result of the different allocations of the workforce across the various cash transfers. They occur because the conditional cash transfer reaches the long-run equilibrium share of employment in agriculture first (see figure 14). That means the farmers who are innovating are, on average, more productive than the farmers in the two unconditional cash transfers, which means they would innovate relatively more. This explains why \(z^C / z^U\) increases in the later stages of developing in figure 13.
them increased access to food with the same number of resources. On the surface this would seem like an improvement because it moves the economy closer to the long-run equilibrium where only a few households work as farmers. However, the general equilibrium effect of this result is to lower household innovation. The food aid lowers the amount of time that households dedicate towards farming, which in turn lowers the return a household receives for innovating. Because the unconditional cash transfer does nothing to lower the cost of innovation, household systematically devote fewer resources to improving their productivity. This lowers the growth rate of output per agricultural worker (as evidenced in panel b). As a result, despite the fact that the economy under $\bar{\xi}$ start with significantly less labor employed in agriculture it actually takes longer for these economies to reach the long-run equilibrium.

5.2.1 Discussion

There is a question of how these results fit into the cash transfer literature as a whole. As mentioned in the introduction, the literature on cash transfers is vast and this section will only provide a brief overview before focusing on a couple of relevant papers. Cash transfers as a whole have been extremely effective. In particular, both conditional and unconditional cash transfers have been shown to be effective at improving outcomes in childhood (Atanasio and Mesnard, 2006; Aguero et al. 2016; Macours et al. 2012; Amarante et al., 2011; Edmonds and Schady, 2012; Fernald and Hidrobo, 2011). In terms of household investment, five papers are particularly relevant. First, Gertler et al. (2012) look at PROGRESSA’s effect on investment and long-term consumption, finding that 26 cents of every peso given to households was invested, which increased long-term consumption by 1.6 pesos. Blattman et al. (2015) look at the ef-
fects of cash transfers on self-employment. The grants were conditional on groups filling out a proposal, however the use of the grant was unsupervised. They found significant increases in business assets, work hours, and earnings. Using unconditional cash transfers, Haushofer and Shapiro (2016) find that lump-sum transfers are more likely to be spent on durable goods, while smaller monthly transfers are more effective at improving food security. Finally, De Mel et al (2008, 2012) provide unconditional grants of cash and equipment to micro-enterprises in Sri Lanka. They find that not only do micro-enterprises have high marginal returns, but also unconditional grants result in better long-run survival rates and higher profits.

While the evidence clearly points to the effectiveness of cash transfers in improving a wide range of outcomes, what is unclear (and what this paper speaks to) is which type of cash transfer is most beneficial. This impact of cash transfers on the model presented in this paper is consistent with the literature listed above as both food security and investment increase with either type of intervention. The model also shows that conditional cash transfers are unambiguously more effective at raising household investment and consumption than unconditional. It turns out that this lines up quite well with limited empirical evidence that does exist. de Brauw and Hoddinott (2011) and Baird et al (2011), among others, show that conditional cash transfer significantly improve educational outcomes relative to unconditional transfers. These findings are not without caveats as Benhassine et al. (2015) show that “conditioning” a cash transfer may be as simple as telling people what the money should be used for. On the theoretical side Mookherjee and Ray (2008) show that unconditional cash transfers create what they call a “welfare magnet” which results in lower human capital investment, income, and consumption, whereas conditional cash transfer have the opposite effect.

As a whole this paper is consistent with both the empirical and theoretical findings in the literature on cash transfers. Due to high marginal utility, unconditional cash transfers are mostly spent on consumption rather than productive investment. This results in worse long-run outcomes for the household as they experience lower income and consumption.

6 Conclusion

This paper presents a model that shows the importance of considering subsistence requirements in the discussion of economic growth in developing countries. Subsistence consumption requirements impact household choices in two ways. First, it limits the amount households can spend on investment and innovation by putting constraints on disposable income. Second, when households only care about consumption net of subsistence, subsistence requirements
increase risk aversion. This paper shows that when households are poor, the disposable income constraints have the greatest effect on household behavior. However, the aggregate impacts of increased risk aversion on economic growth are quite large during the duration of economic development.

In addition to quantifying the impacts of subsistence consumption on household innovation and economic growth, this paper considered the policy implications of these requirements. In particular, section 5 looked at the relative efficiency of conditional and unconditional cash transfers. This investigation yielded three main results. First, conditional cash transfers outperform unconditional cash transfers. Despite the disposable income constraints imposed by subsistence consumption requirements, the added incentive to innovate created by conditional cash transfers resulted in greater rates of household investment. Second, the size of the gap in performance between conditional and unconditional cash transfers depends negatively on the probability of success. In essence, the disposable income constraints were more important for those projects with less uncertainty because risk aversion played a smaller role. This is consistent with the experimental evidence that suggests conditional and unconditional cash transfers have similar rates of effectiveness. These studies showed that people were just as likely to invest in their business with unconditional cash transfers, however these investments tended to be in low-return/low-uncertainty activities (buying more inventories, cattle, etc.), which is consistent with the results of this paper. Finally, large unconditional cash transfers can actually reduce overall investment in agricultural productivity because it reduces the amount of time households spend farming, thus lowering the return to innovation.

This paper opens up several avenues of future research, in particular the role of insurance markets in correcting the effects of decreasing relative risk aversion. While this model cannot speak directly to the issue of moral hazard, it is quite straightforward to see that the introduction of insurance would provide households with the incentives to claim the attempted adoption of a new technology without actually undertaking it. Future work should look at optimal insurance contracts that will improve on the outcomes of households with DRRA preferences without falling victim to moral hazard.

References


A Simple Model

A.1 Aggregate Growth Rate: Proposition 2 Proof

Proof. Household investment in productivity is given by:

\[ z_{it} = \left( \frac{q_{it-1} - \bar{c}}{q_{it-1}} \right) (2\chi - 1) \]

This implies that successful innovators will produce at time \( t \) with productivity:

\[ q^S_{it} = q_{it-1}(1 + z_{it}) = 2\chi q_{it-1} - (2\chi - 1)\bar{c} \tag{36} \]

and unsuccessful innovators will produce at time \( t \) with productivity:

\[ q^F_{it} = q_{it-1}(1 - z_{it}) = 2(1 - \chi) q_{it-1} + (2\chi - 1)\bar{c} \tag{37} \]

I will denote the set of agents who fail to innovate by \( \mathcal{F}_t \) and those who successfully innovate by \( \mathcal{S}_t \). Total output is then given by:

\[
Y_t = \int_0^1 q_{it} di = \int_{i \in \mathcal{S}_t} q^S_{it} di + \int_{i \in \mathcal{F}_t} q^F_{it} di + \int_{i \in \mathcal{S}_t} q^S_{it} di + \int_{i \in \mathcal{F}_t} q^F_{it} di
\]

\[
= 2(1 - \chi) \int_{i \in \mathcal{F}_t} q_{it-1} di + 2\chi \int_{i \in \mathcal{S}_t} q_{it-1} di + (2\chi - 1)\bar{c} \int_{i \in \mathcal{F}_t} di + (2\chi - 1)\bar{c} \int_{i \in \mathcal{S}_t} di
\]

Because being successful or unsuccessful is independent of household productivity, by the law of large numbers the average productivity among each type of household would be the same, i.e. the population average (\( \hat{q} \)):

\[ \hat{q}_{t-1} = \int q y dG_{t-1}(y) \]

Note that because the population has been normalized to 1, the population average of productivities is also total output. This implies that:

\[
\int_{i \in \mathcal{F}_t} q_{it-1} di = (1 - \chi) \hat{q}_{t-1}
\]

\[
\int_{i \in \mathcal{S}_t} q_{it-1} di = \chi \hat{q}_{t-1}
\]

Also note that:

\[
\int_{i \in \mathcal{F}_t} di = (1 - \chi)
\]

\[
\int_{i \in \mathcal{S}_t} di = \chi
\]
Substituting these into the aggregate output equation yields:

\[ Y_t = 2((1 - \chi)^2 + \chi^2) \hat{q}_{t-1} - (2 \chi - 1)^2 \hat{c} \]

Therefore the growth rate of output is:

\[
g_t = \frac{Y_t}{Y_{t-1}} - 1 = \frac{2((1 - \chi)^2 + \chi^2) \hat{q}_{t-1}}{Y_{t-1}} - \frac{(2 \chi - 1)^2 \hat{c}}{Y_{t-1}} - 1
\]

= \left(2\chi - 1\right)^2\left(1 - \frac{\hat{c}}{Y_{t-1}}\right)

where the second line makes use of the fact that \( Y_{t-1} = \hat{q}_{t-1} \)

**A.2 CRRA vs DRRA: Proposition 3 Proof**

*Proof.* Let \( Y_t \) denote output under DRRA utility and \( \hat{Y}_t \) denote output under CRRA utility, then:

\[
\hat{Y}_t - Y_t = \int_0^1 \hat{q}_{it} di - \int_0^1 q_{it} di = \int_0^1 (\hat{q}_{it} - q_{it}) di
\]

= \( \int_{i \in \mathcal{F}_t} q_{it-1}(1 - \hat{z}_{it}) - q_{it-1}(1 - z_{it}) di + \int_{i \in \mathcal{S}_t} q_{it-1}(1 + \hat{z}_{it}) - q_{it-1}(1 + z_{it}) di \)

= \( \int_{i \in \mathcal{F}_t} q_{it-1}(z_{it} - \hat{z}_{it}) di + \int_{i \in \mathcal{S}_t} q_{it-1}(\hat{z}_{it} - z_{it}) di \)

Note that:

\[
z_{it} - \hat{z}_{it} = \begin{cases} 
-2(1 - \chi)\left(\frac{q_{it-1} - \hat{c}}{q_{it-1}}\right) & \text{if } q_{it-1} \leq \hat{q} \\
-(2\chi - 1)\frac{\hat{c}}{q_{it-1}} & \text{if } q_{it-1} > \hat{q}
\end{cases}
\]

\[
\hat{z}_{it} - z_{it} = \begin{cases} 
2(1 - \chi)\left(\frac{q_{it-1} - \hat{c}}{q_{it-1}}\right) & \text{if } q_{it-1} \leq \hat{q} \\
(2\chi - 1)\frac{\hat{c}}{q_{it-1}} & \text{if } q_{it-1} > \hat{q}
\end{cases}
\]

Again applying the law of large numbers we have:

For \( q_{it-1} \leq \hat{q} \):

\[
\int_{i \in \mathcal{F}_t} q_{it-1} di = \int_{i \in \mathcal{F}_t} q_{it-1} di = \int_{\hat{q}}^{\bar{q}} y d\bar{t}_{t-1}(y)
\]

For \( q_{it-1} > \hat{q} \):

\[
\int_{i \in \mathcal{F}_t} q_{it-1} di = \int_{i \in \mathcal{F}_t} q_{it-1} di = \int_{\hat{q}}^{\infty} y d\bar{t}_{t-1}(y)
\]
Substituting these into the above equation yields:

\[ \dot{Y}_t - Y_t = 2(1 - \chi)(2\chi - 1) \int_{\bar{q}}^{\hat{q}} \left( y - \bar{c} \right) dG_{t-1}(y) + (2\chi - 1)^2 (1 - G_{t-1}(\hat{q})) \]

which is unambiguously positive.

\[ \square \]

### B  Cash Transfers: Simple Model

In order to determine which type of cash transfer is most effective, it is important to understand how household investment varies across the probability of success. First, note that both investment functions are strictly increasing and linear in the probability of success. Next it is clear that:

\[ z^C(q, \bar{c}, \zeta, 0.5) > z^U(q, \bar{c}, \zeta, 0.5) = 0 \]

This says that at the lower limit for \( \chi \), a household will invest more in innovation if they received a conditional cash transfer. The upper limit is slightly more ambiguous.

\[ z^C(q, \bar{c}, \zeta, 1) = \frac{(q - \bar{c})}{q - \zeta} \]

\[ z^U(q, \bar{c}, \zeta, 1) = \frac{q - \bar{c} + \zeta}{q} \]

It turns out that \( z^U > z^C \) only if:

\[ \zeta < \frac{\zeta q}{q - \bar{c} + \zeta} \quad (38) \]

If equation (38) does not hold, then the conditional cash transfer is unambiguously more effective at raising household investment. However, assuming that equation (38) holds, then the linearity of the two policy functions implies that there is a unique crossing point at:

\[ \bar{\chi} = \frac{(q - \zeta)(-c\zeta + \zeta\xi + q(\zeta + \xi))}{2\zeta^2(c - q) + 2\xi(q - \zeta)(\zeta + q)} \]

Figure 12 plots the two household investment curves as a function of the probability of success.

The natural question that arises from figure 12 is what happens when the two types of cash transfers net the household the same amount of financial gain. To examine this possibility, the unconditional cash transfer is set to be:

\[ \dot{\zeta} = \zeta * \frac{(q - \bar{c})(q(2\chi - 1) + \zeta)}{q^2 - \zeta^2} \]
In order to determine the impact of restricting the unconditional cash transfer to be equal to the conditional cash transfer, I will examine the difference between the two types of household investment.

\[ z^C(q, \bar{c}, \zeta, \chi) - z^U(q, \hat{c}, \hat{\xi}, \chi) = \frac{4\zeta(1 - \chi)(q - \bar{c})}{q^2 - \zeta^2} > 0 \]

Clearly, the incentives provided by conditional cash transfers result in larger household investment when the financial gains are the same.

C Calibration

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<th>Non-Agriculture Growth Rate</th>
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D Empirical Test
Figure 15: Empirical Test by Country
Figure 16: Trade as a Percentage of GDP by Country