Inequality and the Ability to Aspire*

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Final Version

Abstract

Households with *ex ante* identical preference and ability but heterogeneous wealth decide whether or not to aspire to a common benchmark. The choice depends on the tradeoff between higher utility from wealth accumulation and lower utility from falling short. People choose to be aspirational if they are wealthy enough. This creates a tendency for polarization of aspirations and wealth. Demographic change counteracts it. As the relationship between fertility and household income goes from positive to negative, the non-aspirational poor procreate at a faster rate which, through the aspirational benchmark, brings aspirations within their reach. Not everyone aspires in the long run and wealth and lifetime utility gaps persist if the response to aspirations is strong.

Keywords: Aspirations, Status-seeking, Persistent Inequality, Fertility, Endogenous preference, Preference externality.
JEL Classification: D31, J1

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1 Introduction

This paper is an attempt to understand how aspirational behavior – conceptualized as the urge to do at least as well as others – emerges in a population, adapts to demographic change and, in the process, shapes economic inequality.

In an overlapping generations model with intergenerational altruism, people have *ex ante* identical preferences and ability but differing wealth. They decide whether or not to aspire to the economy-wide average wealth.[1] Aspirations motivates them to accumulate more wealth in the hope of higher consumption and bequests in the future; the cost is utility loss from failing to attain it. Asset-poor people choose not to aspire, and this inability amplifies the advantages of wealth. Specifically, because the wealth distribution influences aspirations which, in turn, influences saving behavior differentially based on who can or cannot aspire, inequality of initial wealth can persist over time.

What effect the preference externality has on wealth dynamics depends on reproductive behavior too. If the aspirational rich procreate at a faster rate, their wealth advantage pushes aspirations further out of reach of the poor, amplifying inequality. If the poor procreate at a faster rate, on the other hand, aspirations becomes more attainable to them. We introduce demographic dynamics through fertility choice subject to the quantity-quality tradeoff.

The fertility of the rich versus the poor depends on two margins. Because aspirations motivates wealth accumulation, it lowers the demand for children and raises intergenerational transfers. All else equal, the rich have a lower fertility propensity due to this. On the other hand, if wages are low, so is the opportunity cost of child-rearing and the rich opt to have more children. The latter margin dominates as long as wages are low enough, and higher fertility of the rich increases divergence between the aspirational rich and non-aspirational poor. Over time, because of productivity growth, wages eventually rise enough to set off a fertility transition. The rich respond by lowering fertility, the poor initially by raising theirs, later by lowering. The wealth gap narrows as more and more of the poor aspire and join the ranks of the wealthy. Inequality of wealth, aspirations and lifetime utilities can, however, persist. Importantly, this occurs despite the demographic forces favoring convergence towards equality.

This paper lies at the intersection of several literatures. Building on a large body of

[1] What we call aspirations is variously referred to as the "rat race", "status seeking", "Keeping up with the Joneses", "envy and pride" in the literature (Hopkins, 2008). We prefer the term aspirations as it makes agents future-oriented (Appadurai, 2004) and pushes them to better their lives. Indeed they choose to engage in it only if it does.

work on other-regarding preferences,[2] we introduce an extensive margin, the choice to be aspirational. While the effect of aspirations on economic behavior is not particularly different from the literature, it is the interaction of that behavior with the extensive margin that yields interesting insights on the dynamics of inequality and aspirational culture. To the long list of factors that have been advanced as causes of persistent inequality,[3] we add aspirations as a potential contributor. This implication is quite at odds with Friedman's (1962) belief that inequality is desirable because it motivates the worse-off to do better.[4]

Our specification of aspirations choice is closely related to Barnett *et al.* (2012). In a static model of consumption-leisure choice and status-seeking with respect to consumption, the authors find that less productive workers opt out of the rat race, amplifying fundamental inequality. In a similar spirit, Genicot and Ray (2017) study individuals who aspire with respect to intergenerational transfers and undertake specific investment to close their aspirations gap. Because they respond more strongly to the gap the closer they are to their aspirations, no investment is made when individuals are far below their aspirations. The result is income polarization. While the outcome here is similar, we allow for family size to affect intergenerational transfers over time and do not rely on non-convexities in the aspirations function. A different approach taken by Dalton *et al.* (2013) assumes aspiration to be internal, an outcome of an individual decision-making. Boundedly-rational individuals do not recognize the feedback loop leading to aspirations failure and persistent poverty. In our model, there is no aspirations failure in the sense of people failing to live up to their potential, though a positive equilibrium aspirations gap for some households is akin to the disappointment of failing to live up to one's goals.

A novel contribution of our work is the role of demography, relatively unexplored in this literature. Two papers, Tournemaine (2008) and Tournemaine and Tsoukis (2010), show that status-conscious households have fewer children and use it to explain the fertility transition as resulting from an *exogenous* shift towards stronger status concerns. A separate literature studies preference formation, including relative concerns, from the point of view of evolutionary fitness. For example, it has been argued that if status-seeking confers an economic advantage, it also confers a reproductive advantage due to which the trait

[2]On the macroeconomics side of this literature, contributions such as de la Croix and Michel (1999), Corneo and Jeanne (1999), Alonso-Carrera *et al.* (2007), García-Peñalosa and Turnovsky (2008), Kawamoto (2009), Moav and Neeman (2010) and Strulik (2013) identify various consequences of exogenous status-seeking for individual and aggregate outcomes.

[3]See Galor and Zeira (1993), Ghatak and Jiang (2002), Mookherjee and Ray (2003), Chakraborty and Das (2005), and Gulati and Ray (2016) for models with capital market frictions, real and pecuniary externalities, human capital and health.

[4]In Allen and Chakraborty's (2018) model of upward-looking aspirations, the poor have a hard time responding to their higher aspirations gap because they lack the flexibility to work longer hours. Besides amplifying fundamental inequality, this has additional welfare effects because of health losses.

spreads through society over time (Fershtman and Weiss, 1998). This assumption of a positive fertility-income gradient is at odds with modern societies where economic advantage does not typically translate into reproductive advantage in terms of family size. Our approach, based on opportunity sets and rational choice, differs fundamentally from the selection-based literature. But the inverse relationship between fertility and income, curiously, works in a similar fashion as it enables more people to become aspirational. In the long-run all households become aspirational only if the response to aspirations is "weak enough."

In a small way, this paper also adds to two other bodies of work. Our model generates an empirically plausible relationship between fertility and income using a margin, the relative importance of labor and non-labor wealth in household budgets, that has not been studied much in the economic demography literature (Galor, 2012, provides a nice overview). The model's cross-sectional and time-series fertility implications and relevance of aspirations for long-term development are studied in a companion paper, Allen and Chakraborty (2022). We show there that cultural change, through aspirations, can replicate the English economic success of the eighteenth and nineteenth centuries as well as its secular decline in the real interest rate. Also relevant to our work is the literature on the disproportionately low saving and bequest propensities of poorer households. One plausible explanation is non-homothetic preferences, for example, bequests-as-luxuries as in Moav (2002). Because poorer households do not aspire and therefore, save or bequeath as much wealth in our model, cross-sectionally it produces a positive association of saving and bequest propensities with wealth.

The paper is organized as follows. After specifying the decision problem faced by households in section 2, section 3 studies the exogenous fertility case to highlight how aspirations formation generates persistent inequality. Section 4 considers decisions under fertility choice and shows how demography acts as a counterweight. Production technologies are specified and the dynamic equilibrium analyzed in section 5. Section 6 concludes.

2 Preferences

The economy is populated by a continuum of intergenerationally altruistic households. They live for three periods: dormant childhood, active youth and retired old age. Households have *ex ante* identical preferences but differ in their initial (inherited) wealth. They choose, in their youth, how many children to have and how much bequest to leave to each of them.

We introduce the possibility of aspirations or status-seeking by assuming people compare how well-off they are relative to a reference wealth level. This reference is taken to be proportional to the average wealth in one's cohort. In particular, a young household has logarithmic preference over it's aspiration gap, $\alpha = \kappa\bar{z}/z$, where z is mid-life wealth and $\bar{z}$ the average wealth of households in the same cohort. For $\kappa > 1$, the household strives to be "better than the average" and for all κ, it derives an ego-rent from exceeding $\kappa\bar{z}_t$ and a disutility from falling below. Fully rational households choose to be aspirational as long as it yields higher utility. This is similar to Becker and Mulligan (1997) where households choose their effective rate of time preference and, especially, Barnett *et al.* (2010) where status is derived with respect to a consumption benchmark.

A household with initial wealth (inheritance) a_t maximizes lifetime utility

$$U_t = \ln c_{1t} + \beta \ln c_{2t+1} + \gamma \left[\theta \ln n_t + (1-\theta) \ln a_{t+1}\right] - \mathcal{I}_t \lambda \ln \alpha_t \tag{1}$$

subject to the two budget constraints

$$c_{1t} + z_t + \delta n_t = (1 - \tau n_t) w_t + a_t \tag{2}$$

$$c_{2t+1} + n_t a_{t+1} = R_{t+1} z_t \tag{3}$$

by choosing $\{c_{1t}, c_{2t+1}, z_t, n_t, b_{t+1}, \mathcal{I}_t\}$. Here $n_t \in [0, 1/\tau]$ is the number of children, δ is the resource and $\tau \in (0,1)$ the time cost per child, and a_{t+1} is the bequest made to each child. The indicator function $\mathcal{I}_t$ takes the value 1 if the household chooses to be aspirational and zero otherwise. The parameter $\lambda > 0$ measures responsiveness to the aspirations gap; more generally one can imagine households choosing λ on a continuum. Each household is endowed with one unit of labor time in youth that earns the competitive wage w_t. Savings are invested on the capital market, earning the gross return R_{t+1}. Households take as given factor prices $\{w_t, R_t\}$ and inheritance $a_t \geq 0$.

Let the cumulative distribution of initial wealth in generation t be $G_t(a)$ which specifies the proportion of households with assets below some a. The economy starts at $t = 0$ with an initial distribution $G_0(a)$ and subsequent distributions evolve based on household behavior.

3 Exogenous Fertility

The dynamics of wealth and aspirations depend on two margins, preference externality and endogenous fertility. By first studying a version of the model with exogenous fertility, we show that the preference externality creates a tendency towards polarization, that is, persistent inequality in wealth and aspirations even though households do not differ intrinsically.

Suppose all agents have $n_t = n$ children. Without loss of generality, set $n = 1$ and child rearing costs $\delta = \tau = 0$. A generation-t household now solves the decision problem

$$\max\ U_t = \ln c_{1t} + \beta \ln c_{2t+1} + \gamma(1-\theta)\ln a_{t+1} - \mathcal{I}_t \lambda \ln \alpha_t \tag{1'}$$

subject to

$$c_{1t} + z_t = w_t + a_t, \tag{2'}$$
$$c_{2t+1} + a_{t+1} = R_{t+1} z_t. \tag{3'}$$

For expositional convenience, let's analyze decisions sequentially. We first study economic choices conditional on aspirations choice, then ask which aspirations choice yields higher lifetime utility.

Suppose the household chooses not to be aspirational, $\mathcal{I}_t = 0$. Label all such households type N. Their choices are linear in working-life wealth, $w_t + a_t$

$$c_{1t}^N = \frac{1}{1+\beta+\gamma(1-\theta)}(w_t + a_t) \equiv \mu_{c1}^N (w_t + a_t) \tag{4}$$
$$c_{2t+1}^N = \frac{\beta}{1+\beta+\gamma(1-\theta)} R_{t+1}(w_t + a_t) \equiv \mu_{c2}^N R_{t+1}(w_t + a_t) \tag{5}$$
$$z_t^N = \frac{\beta+\gamma(1-\theta)}{1+\beta+\gamma(1-\theta)}(w_t + a_t) \equiv \mu_z^N (w_t + a_t) \tag{6}$$
$$a_{t+1}^N = \frac{\gamma(1-\theta)}{1+\beta+\gamma(1-\theta)} R_{t+1}(w_t + a_t) \equiv \mu_b^N R_{t+1}(w_t + a_t) \tag{7}$$

Similarly, for an aspirational household labeled type A, decisions under $\mathcal{I}_t = 1$ are

$$c_{1t}^A = \frac{1}{1+\beta+\gamma(1-\theta)+\lambda}(w_t + a_t) \equiv \mu_{c1}^A (w_t + a_t) \tag{8}$$
$$c_{2t+1}^A = \frac{\beta}{\beta+\gamma(1-\theta)} \frac{\beta+\gamma(1-\theta)+\lambda}{1+\beta+\gamma(1-\theta)+\lambda} R_{t+1}(w_t + a_t) \equiv \mu_{c2}^A R_{t+1}(w_t + a_t) \tag{9}$$
$$z_t^A = \frac{\beta+\gamma(1-\theta)+\lambda}{1+\beta+\gamma(1-\theta)+\lambda}(w_t + a_t) \equiv \mu_z^A (w_t + a_t) \tag{10}$$
$$a_{t+1}^A = \frac{\gamma(1-\theta)}{\beta+\gamma(1-\theta)} \frac{\beta+\gamma(1-\theta)+\lambda}{1+\beta+\gamma(1-\theta)+\lambda} R_{t+1}(w_t + a_t) \equiv \mu_b^A R_{t+1}(w_t + a_t) \tag{11}$$

The proportionality constants now depend on λ.

Observe that aspirational households are strongly motivated to accumulate wealth

($\mu_z^A > \mu_z^N$) because they have the additional incentive to narrow their aspirations gap. Non-aspirational households, on the other hand, have a higher propensity to consume early in life ($\mu_{c1}^N > \mu_{c1}^A$) since savings has no value beyond future consumption and bequest. Consequently they have a lower bequest propensity ($\mu_b^N < \mu_b^A$) too.[5]

3.1 The choice to be aspirational

Rational households adopt whichever aspirational behavior provides them higher lifetime utility. The indirect utilities of a household with initial wealth a_t are

$$V_t^N(a_t) = [1+\beta+\gamma(1-\theta)]\ln(w_t+a_t) + \left[\ln\mu_{c1}^N + \beta\ln\mu_{c2}^N + \gamma(1-\theta)\ln\mu_b^N\right] + [\beta+\gamma(1-\theta)]\ln R_{t+1}.$$

for $\mathcal{I}_t = 0$, and

$$V_t^A(a_t) = [1+\beta+\gamma(1-\theta)+\lambda]\ln(w_t+a_t) + \left[\ln\mu_{c1}^A + \beta\ln\mu_{c2}^A + \gamma(1-\theta)\ln\mu_b^A + \lambda\ln\mu_z^A\right] + [\beta+\gamma(1-\theta)]\ln R_{t+1} - \lambda\ln(\kappa\bar{z}_t)$$

$\mathcal{I}_t = 1$. It is straightforward to show that both are increasing in initial wealth with V_t^A increasing faster since aspirational households accumulate more wealth. Since V_t^A entails a disutility from falling short of $\kappa\bar{z}_t$, we have $V_t^A(0) < V_t^N(0)$.

Proposition 1. *The choice to be aspirational depends only on initial wealth. Households with initial wealth*

$$a_t \geq \Omega\kappa\bar{z}_t - w_t \equiv \hat{a}_t \tag{12}$$

where $\Omega = (\Omega_N/\Omega_A)^{1/\lambda}$, $\Omega_1 \equiv \mu_{c1}^N(\mu_{c2}^N)^\beta(\mu_b^N)^{\gamma(1-\theta)}$ *and* $\Omega_A \equiv \mu_{c1}^A(\mu_{c2}^A)^\beta(\mu_b^A)^{\gamma(1-\theta)}(\mu_z^A)^\lambda$*, choose to be aspirational,* $\mathcal{I}_t = 1$*. Those below* $\hat{a}_t$ *choose to be non-aspirational,* $\mathcal{I}_t = 0$*.*

Proof. Follows from Figure 1 where $V_t^A(a_t) \geq V_t^N(a_t)$ for $a_t \geq \hat{a}_t$. □

That asset-poor households choose to be non-aspirational is similar to Barnett *et al.* (2010) where less productive, poorer, households are unable to keep up with the rat race and "opt out".

[5] It is not essential for these results that aspirations be with respect to one's own position in society. Households may derive pleasure from seeing their children do better than others as in Genicot and Ray (2017). Appendix A shows that aspirations with respect to bequests produces similar behavior.

It is, however, essential to the model that households aspire with respect to a long-term asset, be it financial or human capital or child quality, that drives intergenerational progress. Consumption-based aspirations in the OLG framework, as in Alonso-Carerra *et al.* (2007), discourage intergenerational wealth accumulation.

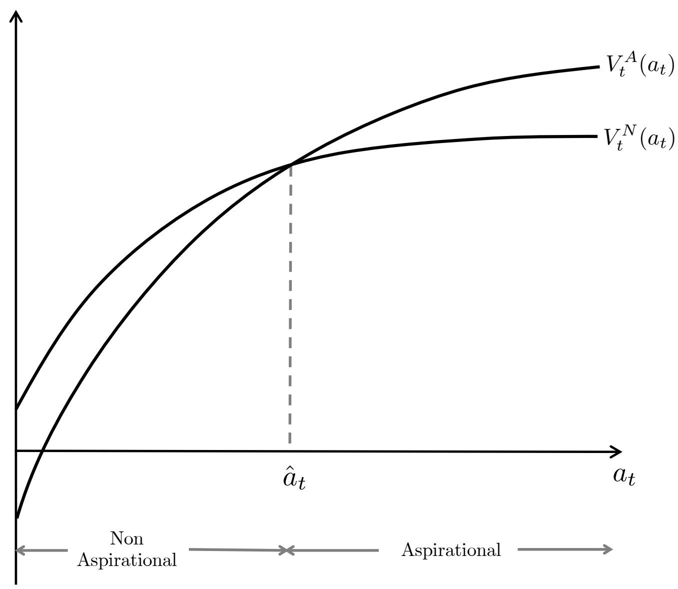

Figure 1: Aspirations Decision

3.2 Dynamics

Aspirations introduces an interdependence between the threshold wealth $\hat{a}_t$ that determines the opting-in decision and aggregate saving $\bar{z}_t$ through equation (12). The latter depends on the saving behavior of both aspirational and non-aspirational households:

$$\bar{z}_t = \int_0^{\hat{a}_t} z_t^N dG_t + \int_{\hat{a}_t}^{\infty} z_t^A dG_t = \int_0^{\hat{a}_t} \mu_z^N(\omega + a_t) dG_t + \int_{\hat{a}_t}^{\infty} \mu_z^A(\omega + a_t) dG_t. \tag{13}$$

which, of course, depends on the decision to opt in, that is $\hat{a}_t$. It is this interdependence that generates persistent inequality over time without any inherent differences across households.

Consider the intergenerational wealth dynamics implied by equations (7) and (11). Suppose prices are constant $w_t = \omega$ and $R_t = \rho$ for all t. Then wealth dynamics are specified by the piece-wise linear difference equation

$$a_{t+1} = \begin{cases} \mu_b^N \rho(\omega + a_t), & \text{if } a_t < \hat{a}_t \\ \mu_b^A \rho(\omega + a_t), & \text{if } a_t \geq \hat{a}_t \end{cases} \tag{14}$$

with $\mu_b^A > \mu_b^N$, $\hat{a}_t$ defined by (12) and $G_0(a)$ given. Assume that $\mu_b^A \rho < 1$ so that dynastic wealth is bounded. The fixed points a_1^* and $a_2^* > a_1^*$ of the two pieces of (14) are then

$$\begin{aligned} a_1^* &= \frac{\mu_b^N \rho}{1-\mu_b^N \rho}\omega \equiv (\xi_1 - 1)\omega, \\ a_2^* &= \frac{\mu_b^A \rho}{1-\mu_b^A \rho}\omega \equiv (\xi_2 - 1)\omega. \end{aligned} \tag{15}$$

The stationary wealth distribution can be of three types. Either all households converge to a_1^*, or all converge to a_2^*, or there is polarization with positive mass of households at each of a_1^* and a_2^*. Under what conditions do polarization in wealth and aspirations result instead of unconditional convergence to a_1^* or a_2^*?

To get an intuitive understanding, suppose we denote the fraction of a cohort born to non-aspirational parents by ψ.[6] Suppose the initial wealth distribution G_0 is discrete: initial wealth is either a_{10} or $a_{20} > a_{10}$ with proportions of young households $\psi \in (0, 1)$ and $1 - \psi$ respectively. Figure 2 illustrates the wealth dynamics for $t = 0$ and 1. Given a_{10}, a_{20} and ψ, suppose the decision threshold is $\hat{a}_0 > a_{10}$ in the left panel of the figure (black phaselines). Households that started with wealth a_{10} become non-aspirational, leaving bequest of a_{11} to their offspring. Aspirational ones leave a_{21}. Since saving is increasing in inherited wealth, saving by both type of households in $t = 1$ rises and the wealth threshold $\hat{a}_1$ moves up. As drawn on the right panel, this increase is less than proportionate to the increase in mean saving – for instance if $\kappa\Omega < 1$ in (12). Hence poorer households find themselves above the threshold wealth at $t = 1$. These households now become aspirational as a result of which their wealth accumulation is governed by the upper phase line just like wealthier households. Since aspirational households have a higher bequest and saving propensity, it is likely that at $t = 2$, $\hat{a}_2 < a_{12}$. When that happens, there is convergence in aspirations, saving and bequest. Subsequently both types of households asymptotically converge to a_2^*.

Fig 2 also shows a different outcome using gray phaselines. Suppose we sufficiently lowered the population share of the poor from ψ to ψ' such that $\hat{a}_0' > a_1^*$. The right panel shows that, since the average is more sensitive to the wealth of richer households, at $t = 1$, a_{11} now falls short of the threshold $\hat{a}_1'$. It is more likely here the poor never catch up. Of course the relative population shares of the rich and the poor and their initial wealth levels are not the only cause of such polarization. That possibility depends also on a_2^*/a_1^* which, in turn depends on how strongly people respond to aspirations, that is, λ. For relatively low λ, convergence towards equality is more likely.

[6]Because it is not central to the polarization result, we are taking a short-cut by assuming ψ is the same as the fraction of the cohort that *chooses* not to be aspirational. This may not be true as shown in Section 5.

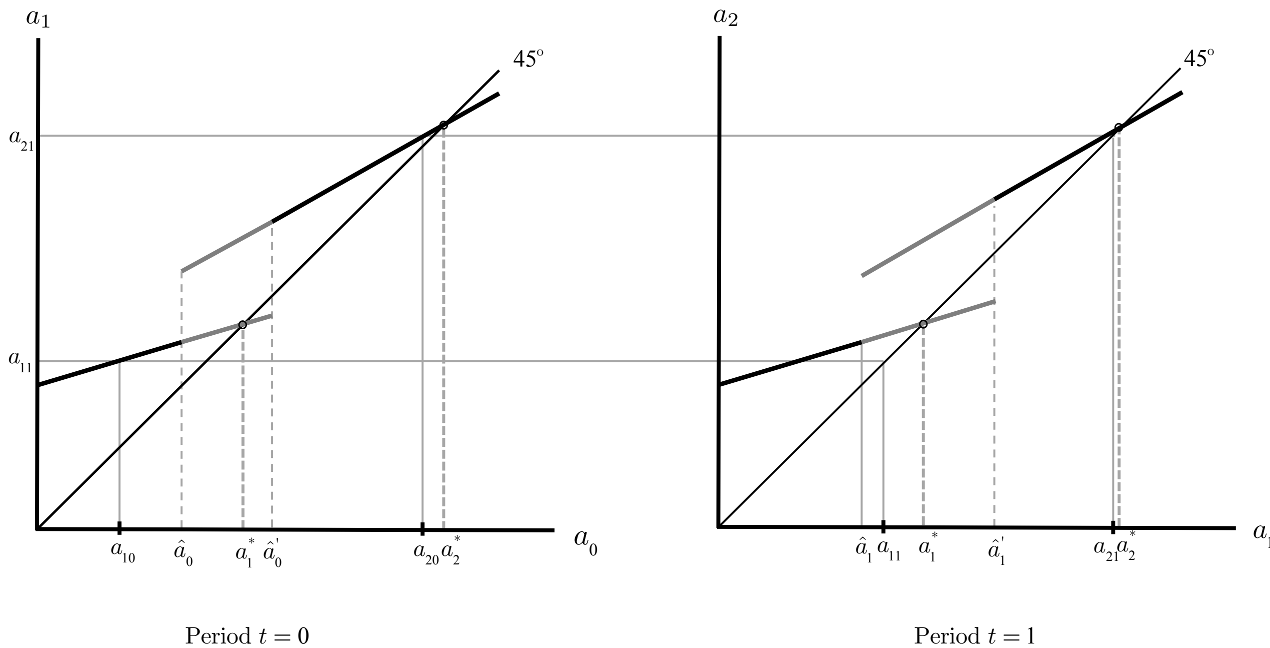

Figure 2: Unconditional and Conditional Convergence under Exogenous Fertility

Proposition 2 formally specifies conditions under which the steady-state distribution is bimodal. We provide a numerical illustration below.

Proposition 2. *The dynamics of wealth following equation* (14) *contains two locally stable steady states* $\{a_1^*, a_2^*\}$ *as long as*

$$\frac{\xi_2}{\xi_1} > \max\left\{\frac{1 - \Omega\kappa\psi\mu_z^N}{\Omega\kappa(1-\psi)\mu_z^A}, \frac{\Omega\kappa\psi\mu_z^N}{1 - \Omega\kappa(1-\psi)\mu_z^A}\right\}. \tag{16}$$

Proof. See Appendix B. □

A Numerical Example

Suppose $\kappa = 1$, $\beta = 0.37$, $\gamma = 0.69$, $\theta = 0.645$ and $\lambda = 0.5$. Figure 3 illustrates three possibilities in line with the qualitative dynamics of Figure 2.[7] For $\psi = 0.1$ (low), too few households with wealth level a_{10} means $\hat{a} > a_2^*$ and everyone converges to the non-aspirational wealth a_1^*. For $\psi = 0.9$ (high), on the other hand, $\hat{a} < a_1^*$ and everyone becomes rich and aspirational in the long run. The intermediate case, $\psi = 0.5$ (medium),

[7]A sufficient proportion of poor non-aspirational households is necessary for the rich to behave aspirationally.

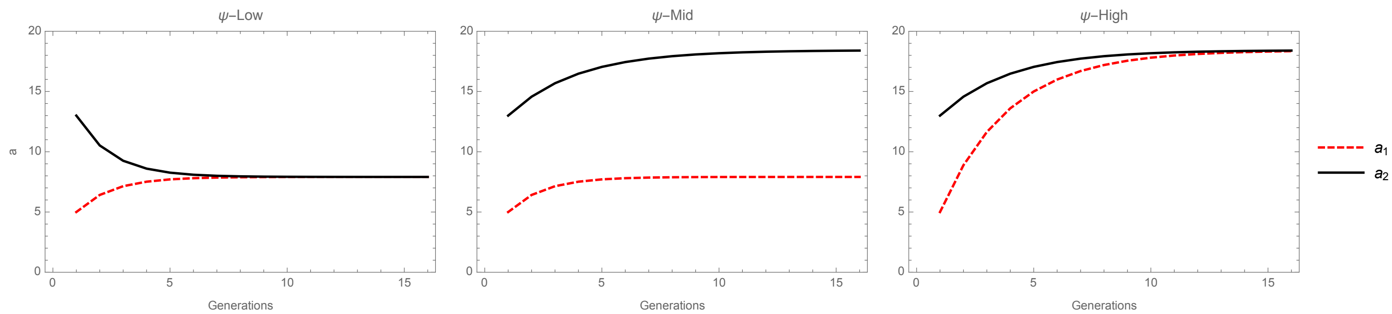

Figure 3: Asymptotically stable unimodal and bimodal wealth distributions

satisfies (16) – the two groups conditionally converge to different steady-state wealth and aspiration levels.

Figure 4 shows, by entertaining alternative values of (ψ, λ), that the parameter space satisfying restriction (16) is fairly large in principle. The solid black line plots ξ_2/ξ_1 while each of the dotted lines, for different values of ψ, plot the right-hand side of (16). The range of λ values for which polarization occurs – whenever the solid black line lies above the dotted line – increases the lower is ψ. For example, consider the dashed line corresponding to $\psi = 0.9$. For all values of λ roughly less than 1, this dashed line lies above the solid line, which implies a long-run unimodal wealth distribution in which everyone is aspirational. In contrast, when $\psi = 0.5$, the dashed line is below the solid line for all values of λ implying a bimodal wealth distribution and a long-run outcome where there are both types of households. The fewer poorer households we have, the higher is the aspirational wealth cutoff $\hat{a}$ and the more likely it is for households at the bottom to be non-aspirational and poor.

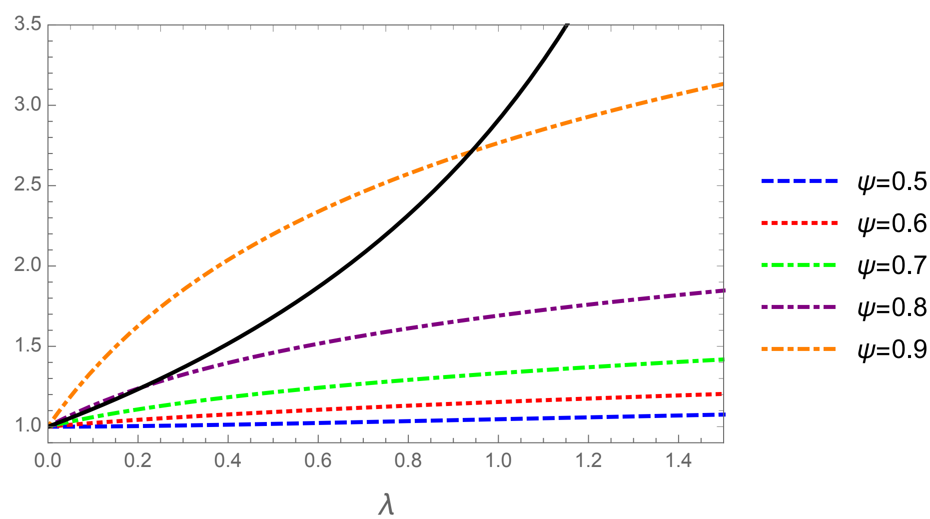

Figure 4: (λ, ψ) combinations that lead to bimodal stationary wealth distribution

3.3 Is Aspirations Evolutionarily Stable?

An interesting question raised by evolutionary models of preference formation is if a particular population trait that confers survival (or economic) advantage is evolutionarily stable. Alger and Weibull (2019) survey some examples of this. Closest to our paper is Fershtman and Weiss' (1998) model of status-conscious agents. In a Prisoner's Dilemma game, agents are born one of two types, those who care about status and those who do not. Social status depends on an agent's effort relative to average effort, and utility depends on monetary returns, payoffs from cooperation and deviation, and gains from social status. Fershtman and Weiss find that for moderate values of the marginal effect of status, an equilibrium where the entire population is socially minded is evolutionarily stable. Central to this result is the assumption that the fraction of people of a given type increases mechanically through Darwinian replicator dynamics if their monetary payoff exceeds the average payoff in the population, that is, their type brings economic success.

The dynamics discussed above shows such an outcome is possible from the extensive margin alone – each household choosing their type rather than being born with it – but there is no guarantee that it will. In other words, when polarization of wealth occurs, not everyone ends up aspirational. Convergence to universal aspirations occurs only for relatively low values of λ and κ that govern the infra-marginal and marginal benefits of aspirational behavior, respectively. The interesting aspect of endogenous fertility is that it makes the replication rate of each type fundamentally different from Darwinian replicator dynamics, which can either amplify or attenuate these margins depending on the relationship between fertility and household wealth.

Three points to note before we move on. First, as is evident from Figure 1, household preferences are given by the non-convex upper envelope of the two indirect utility functions. This non-convexity is not essential to polarization. Barnett *et al.* (2010) convexify preferences by allowing households to buy lotteries and show that not all the poor do so, and those who do not remain non-aspirational.

Secondly, logarithmic preference is not essential either. Persistence stems entirely from the dependence of preferences on an aggregate variable. As long as households valued their holding of some asset – financial wealth, human capital, bequest – relative to everyone else, the preference externality would generate polarization under a strong aspirational motive.

Thirdly, even though saving and bequest functions are concave at the household level, the aggregate functions are not. This comes from the discontinuity in aspirational behavior at $\hat{a}_t$. There is well-documented evidence of the poor having lower saving and bequest propensities (for example, Dynan *et al.*, 2004) that has been explained using non-homothetic preferences at the individual level (Moav, 2002, Moav and Neeman, 2012). Here household-level preferences are homothetic conditional on aspirational behavior. The non-convexity of saving and bequest functions arise cross-sectionally from other-regarding *endogenous* preference formation.

4 Endogenous Fertility

We return to the original decision problem, maximizing (1) subject to (2) and (3). Non-aspirational (type N) households now choose

$$z_t = \frac{\beta + \gamma(1-\theta)}{1+\beta+\gamma\theta}(w_t + a_t) \equiv \sigma_z^N (w_t + a_t), \tag{17}$$

$$n_t = \frac{\gamma(2\theta-1)}{1+\beta+\gamma\theta}\left[\frac{w_t + a_t}{\tau w_t + \delta}\right] \equiv \sigma_n^N \left[\frac{w_t + a_t}{\tau w_t + \delta}\right], \tag{18}$$

$$a_{t+1} = \frac{1-\theta}{2\theta-1}R_{t+1}(\tau w_t + \delta) \equiv \sigma_b^N R_{t+1}(\tau w_t + \delta), \tag{19}$$

$$c_{1t} = \left[1 - \sigma_z^N - \sigma_n^N\right](w_t + a_t) \equiv \sigma_{c1}^N (w_t + a_t), \tag{20}$$

$$c_{2t+1} = \left[\sigma_z^N - \sigma_n^N \sigma_b^N\right] R_{t+1}(w_t + a_t) \equiv \sigma_{c2}^N R_{t+1}(w_t + a_t), \tag{21}$$

and aspirational households (type A) choose

$$z_t = \frac{\beta + \gamma(1-\theta) + \lambda}{1+\beta+\gamma\theta+\lambda}(w_t + a_t) \equiv \sigma_z^A (w_t + a_t), \tag{22}$$

$$n_t = \frac{\gamma(2\theta-1)}{1+\beta+\lambda+\gamma\theta}\left[\frac{w_t + a_t}{\tau w_t + \delta}\right] \equiv \sigma_n^A \left[\frac{w_t + a_t}{\tau w_t + \delta}\right], \tag{23}$$

$$b_{t+1} = \frac{1-\theta}{2\theta-1}\left[1 + \frac{\lambda}{\beta + \gamma(1-\theta)}\right] R_{t+1}(\tau w_t + \delta) \equiv \sigma_b^A R_{t+1}(\tau w_t + \delta), \tag{24}$$

$$c_{1t} = \left[1 - \sigma_z^A - \sigma_n^A\right](w_t + a_t) \equiv \sigma_{c1}^A (w_t + a_t), \tag{25}$$

$$c_{2t+1} = \left[\sigma_z^A - \sigma_n^A \sigma_b^A\right] R_{t+1}(w_t + a_t) \equiv \sigma_{c2}^A R_{t+1}(w_t + a_t). \tag{26}$$

Positive bequest requires that

$$\theta > 1/2 \tag{A1}$$

which also ensures that the second order conditions are satisfied.

Aspirations now works through two additional margins. First, aspirational households allocate more towards wealth accumulation partly by conserving on child rearing costs: for a given wealth a, they have fewer children as $\sigma_n^A < \sigma_n^N$. Of course, aspirational households are also expected to have higher a via intergenerational transfers. Hence, *ex ante* it is unclear whether the rich have more or fewer children than the poor. They have more as long as

$$a_{2t} > (1+\lambda_n)a_{1t} + \lambda_n w_t \tag{27}$$

from (18) and (23), defining $\lambda_n \equiv \lambda/(1+\beta+\gamma\theta)$. Amplifying this are intergenerational transfers. Since aspirational households accumulate wealth faster ($\sigma_z^A > \sigma_z^N$), they also have a higher bequest propensity, $\sigma_b^A > \sigma_b^N > 1$. Bequest per child is increasing in the cost per child, $\tau w_t + \delta$, the familiar quantity-quality tradeoff.

Finally, the interaction of aspirations choice with fertility choice over time depends, in part, on how fertility responds to the wage rate. As long as $a < \delta/\tau$, fertility increases with labor income (see section 5 below). This will be useful in producing empirically relevant fertility behavior over time.

4.1 The choice to be aspirational

As before we compare indirect utilities to arrive at this decision. The indirect utility functions of a non-aspirational household is

$$\begin{aligned} V_t^N(a_t) &= (1+\beta+\gamma\theta)\ln(w_t+a_t) - \gamma(2\theta-1)\ln(\tau w_t+\delta) \\ &+ \ln\left[\sigma_{c1}^N(\sigma_{c2}^N)^\beta(\sigma_n^N)^{\gamma\theta}(\sigma_b^N)^{\gamma(1-\theta)}\right] + [\beta+\gamma(1-\theta)]\ln R_{t+1}. \end{aligned} \tag{28}$$

and of an aspirational household

$$\begin{aligned} V_t^A(a_t) &= (1+\beta+\gamma\theta+\lambda)\ln(w_t+a_t) - \gamma(2\theta-1)\ln(\tau w_t+\delta) \\ &+ \ln\left[\sigma_{c1}^A(\sigma_{c2}^A)^\beta(\sigma_n^A)^{\gamma\theta}(\sigma_b^A)^{\gamma(1-\theta)}(\sigma_z^A)^\lambda\right] + [\beta+\gamma(1-\theta)]\ln R_{t+1} - \lambda\ln(\kappa\bar{z}_t). \end{aligned} \tag{29}$$

Both are increasing in inherited wealth a_t, with V_t^A rising faster than V_t^N and $V_t^N(0) < V_t^A(0)$. The household chooses to be aspirational as long as $V_t^A(a_t) \geq V_t^N(a_t)$, or

$$a_t \geq \kappa\Phi\bar{z}_t - w_t \equiv \hat{a}_t, \tag{30}$$

where $\Phi_1 \equiv \sigma_{c1}^N(\sigma_{c2}^N)^\beta(\sigma_n^N)^{\gamma\theta}(\sigma_b^N)^{\gamma(1-\theta)}$, $\Phi_2 \equiv \sigma_{c1}^A(\sigma_{c2}^A)^\beta(\sigma_n^A)^{\gamma\theta}(\sigma_b^A)^{\gamma(1-\theta)}(\sigma_z^A)^\lambda$ and $\Phi \equiv (\Phi_1/\Phi_2)^{1/\lambda}$. Qualitatively this is no different from Proposition 1: poorer households have a harder time narrowing their wealth relative to the aspirational benchmark and, rather than suffer from falling significantly short, choose not to aspire.

4.2 The role of demography

We saw before that the preference externality creates a tendency for polarization. Endogenous fertility adds a wrinkle. If the aspirational rich are more fertile, as replicator dynamics á la Fershtman and Weiss (1998) assumes by positively correlating reproductive success with economic success, it amplifies the divergence between the two types. Faster wealth accumulation by aspirational/richer households makes it harder for poorer households to be aspirational. But over time, as more and more of the population emerge from aspirational/richer families, aspirations becomes the predominant type. In the limit, everyone is aspirational and enjoys the same standard of living. Fertility, in this case, undoes the tendency for polarization.

Conversely, if the aspirational rich are less fertile, the evolutionary stability of aspirations is not guaranteed. As poor/non-aspirational households become more numerous, all else equal, the population gets less and less aspirational. But all else is not equal. The rising frequency of poorer households lowers average wealth to which aspirations are benchmarked. This makes aspirations more attainable to the poor. Whether or not this counteracting force can make the entire population aspirational in the long-run remains to be seen.

Historical economic and demographic transitions tell us that the relationship between fertility and household income is non-monotonic. Over time, as economies have prospered, their total fertility rates have fallen (Galor, 2012). Cross-sectionally, the rich in pre-modern societies had more surviving children than the poor, possibly more childbirths too (Clark, 2007, Clark and Cummins, 2014). With sustained economic progress, that positive relationship between household income and fertility reversed in all developed societies; many developing ones too are undergoing a similar change.

The model can produce such a fertility reversal, in the aggregate and cross-section, if wages grow over time. Rewrite the budget constraint (2) as

$$c_{1t} + z_t + (\delta + \tau w_t)n_t = w_t + a_t \equiv \tilde{a}_t$$

and let's call $\tilde{a}$ potential wealth. A change in the wage rate has three effects on fertility behavior. Through the opportunity cost margin, higher w makes children more expensive and lowers fertility demand. Higher cost per child creates, at the same time, the familiar real (or pure) income effect that lowers fertility demand. Counteracting these is the wealth effect: higher w raises potential wealth $\tilde{a}$ and fertility demand. Straightforward differentiation of (18) or (23) tells us that $dn/dw < 0$ as long as $a > \delta/\tau$, that is, the substitution effect dominates the full income effect for wealth levels above δ/τ. The wealth level matters because a amplifies the pure income effect. Real wealth, $\tilde{a}/w = 1 + a/w$, falls more at higher values of a.[8]

The non-monotonic effect of w on n has two implications for population dynamics. If all households in a cohort inherit less than δ/τ, they all respond to higher w by raising fertility. Secondly, if wages are growing over time, inheritances will too and richer households cross the wealth threshold δ/τ sooner than poorer ones at which point they start reducing fertility. The same dynamics eventually pushes poorer households over the δ/τ wealth threshold and they start reducing fertility. Their fertility rate, however, will be higher than the rich as long as they choose not to aspire.[9]

The next section shows that this fertility change causes the evolution of aspirations to be non-monotonic. The share of population that is aspirational rises initially when the rich have more children, starts falling as the fertility behavior of the rich change, and then rises again as aspirations become attainable for poorer households with higher fertility. In the long run, aspirations may or may not be universally shared by the population.

5 The Evolution of Aspirations

5.1 Technology and Factor Prices

A unique final good, whose price is normalized to unity at every t, is produced using a constant returns to scale technology that combines labor supply of the young with physical capital owned by the old. The final goods sector is perfectly competitive with labor and capital earning their corresponding marginal products. The depreciation rate of capital is hundred percent.

The economy has access to two CRS technologies, *Malthusian* (M) and *Solovian* (S),

$$Y_t^M = F_M(L_t, K_t) = \omega L_t + \rho K_t,$$
$$Y_t^S = F_S(A_t L_t, K_t) = \omega A_t L_t + \rho K_t$$

where L_t denotes aggregate labor input and K_t aggregate capital. For the Solovian technology, labor productivity grows exogenously at the rate $g > 0$, $A_t = (1+g)^t A_0$ starting from $0 < A_0 < 1$, whereas for the Malthusian technology it is constant at unity.

[8]This channel is absent in models of household fertility where only human capital h is intergenerationally transferred, and therefore, a wage change has no effect on the real purchasing power of "potential income" wh/w.

[9]Note that the cause of the fertility transition, the relative intensity of substitution and full income effects, are independent of aspirations choice.

Since both technologies produce the same good and they differ in labor productivity alone, in equilibrium only one – that which produces higher output for a given bundle of inputs – will be used. Since $A_0 < 1$, the Solovian technology starts from a lower level of labor productivity and the Malthusian technology is used as long as $Y_t^M/L_t > Y_t^S/L_t$, that is, $t < \ln[1/A_0]/\ln(1+g) \equiv T$. After T, the economy switches completely to the Solovian technology.

The equilibrium wage per unit of labor as determined by competitive factor and output markets depends on the technology in use

$$w_t = \begin{cases} \omega, & \text{for } t \leq T, \\ \omega A_t, & \text{for } t > T, \end{cases} \tag{31}$$

growing at the rate g per generation after T, while the equilibrium return to capital is $R_t = \rho \quad \forall t \geq 0$ no matter which technology is in use. The choice of linear production functions imply there is no feedback from factor accumulation to factor prices. By avoiding pecuniary externalities, we are able to isolate the effect of the preference externality on wealth and population dynamics.[10]

5.2 Demography

Let the measure of young households, size of cohort-t, be $\mathcal{N}_t$. It consists of $\mathcal{N}_t^N$ households who were born into non-aspirational households and $\mathcal{N}_t^A = \mathcal{N}_t - \mathcal{N}_t^N$ born into aspirational ones. In equilibrium, households may choose a different aspirations type than their parents.

Let χ_t^{ij} be the fraction of $\mathcal{N}_t^i$ households who choose to be type j. For instance, among households born to non-aspirational parents (type N), χ_t^{NA} corresponds to the fraction who choose to be aspirational (type A) and $\chi_t^{NN} = 1 - \chi_t^{NA}$ corresponds to the fraction who choose to be non-aspirational (type N). There are two possibilities: (I) $\chi_t^{NA} > 0$, $\chi_t^{AN} = 0$, and (II) $\chi_t^{AN} > 0$, $\chi_t^{NA} = 0$. We will restrict ourselves to (I) to simplify the exposition as this anticipates the numerical example below. Appendix C describes the more general case.

Accordingly we adopt the simpler notation χ for χ^{NA}. Then, corresponding to the three

[10]See Allen and Chakraborty (2021) for the more general case and endogenous productivity growth.

types of households, we have the fertility rates[11]

$$n_t^{NN} = \sigma_n^N \left[\frac{w_t + a_{1t}}{\tau w_t + \delta} \right], \quad n_t^{NA} = \sigma_n^A \left[\frac{w_t + a_{1t}}{\tau w_t + \delta} \right], \quad n_t^{AA} = \sigma_n^A \left[\frac{w_t + a_{2t}}{\tau w_t + \delta} \right],$$

using the notation n_t^{ii}, $i, j \in \{N, A\}$, where i denotes parental type and j denotes child's type. We also have

$$\mathcal{N}_{t+1}^N = (1-\chi_t) n_t^{NN} \mathcal{N}_t^N, \quad \mathcal{N}_{t+1}^A = \chi_t n_t^{NA} \mathcal{N}_t^N + n_t^{AA} \mathcal{N}_t^A.$$

The proportion of households, $\psi_t \equiv \mathcal{N}_t^N / \mathcal{N}_t$, born into non-aspirational households evolves according to

$$\psi_{t+1} = \frac{(1-\chi_t) n_t^{NN} \psi_t}{\bar{n}_t} \tag{32}$$

where

$$\bar{n}_t \equiv \frac{\mathcal{N}_{t+1}}{\mathcal{N}_t} = \left[(1-\chi_t)\, n_t^{NN} + \chi_t n_t^{NA} \right] \psi_t + n_t^{AA} (1-\psi_t) \tag{33}$$

is the total fertility rate.

How is χ determined? It depends on the wealth threshold $\hat{a}$. Based on the savings of the three types of households, $z_t^{NN} = \sigma_z^N (w_t + a_{1t})$, $z_t^{AA} = \sigma_z^A (w_t + a_{1t})$, and $z_t^{AN} = \sigma_z^N (w_t + a_{2t})$, the average savings per cohort-t household is

$$\bar{z}_t = \left[(1-\chi_t) z_t^{NN} + \chi_t z_t^{NA} \right] \psi_t + z_t^{AA} (1-\psi_t).$$

Hence, from (30), the wealth cutoff is

$$\hat{a}_t = \kappa \Phi \left[\left\{ (1-\chi_t) z_t^{NN} + \chi_t z_t^{NA} \right\} \psi_t + z_t^{AA} (1-\psi_t) \right] - w_t. \tag{34}$$

If $\hat{a}_t > a_{1t}$ under $\chi_t = 0$, none of the households born to non-aspirational parents switch type. Conversely, when $\hat{a}_t < a_{1t}$ under $\chi_t = 1$, every one of them switches from their parent's type.

An interesting case is $\hat{a}_t = a_{1t}$ for $\chi_t \in (0, 1)$. Were all households born into non-aspirational households to become aspirational, wealth accumulation would rise so much (e.g. relatively high λ *ceteris paribus*) that $\hat{a}_t > a_{1t}$. If none of them choose to become aspirational, on the other hand, we would have $\hat{a}_t < a_{1t}$. Hence, while a household born to a non-aspirational parent would be strictly better off by unilaterally becoming aspirational,

[11] Fertility rates n^{AA} and n^{NA} of the two types of aspirational households – those born into aspirational households and those not – converge in one generation since they make identical bequests per child; see equations (19) and (24).

the simultaneous decision to be aspirational by all such households will not be optimal. The only possible equilibrium is when these households are indifferent between the two options, that is, $\hat{a}_t = a_{1t}$. The corresponding value of χ_t – let's label it $\hat{\chi}_t$ – is obtained by setting $\hat{a}_t = a_{1t}$ which, using equations (19) and (6), leads to

$$\hat{\chi}_t = \frac{\kappa\Phi\psi_t\left[\sigma_z^N(w_t + a_{1t}) - \sigma_z^A(w_t + a_{2t})\right] + \kappa\Phi\sigma_z^A(w_t + a_{2t}) - (w_t + a_{1t})}{\kappa\Phi\psi_t(w_t + a_{1t})(\sigma_z^N - \sigma_z^A)}.$$

Hence, equilibrium χ_t is more precisely specified as

$$\chi_t = \begin{cases} 0, & \text{if } \hat{\chi}_t < 0 \\ \hat{\chi}_t, & \text{if } 0 < \hat{\chi}_t < 1 \\ 1, & \text{if } \hat{\chi}_t > 1. \end{cases} \tag{35}$$

The proportion of households that are non-aspirational is $\hat{\psi}_t = (1 - \chi_t)\,\psi_t$ which is lower than the proportion born to non-aspirational households whenever $\chi_t > 0$.

We can now define aggregate labor supply as

$$L_t = \left[\left(1 - \tau n_t^{NN}\right)(1 - \chi_t) + \left(1 - \tau n_t^{AN}\right)\chi_t\right]\psi_t\mathcal{N}_t + \left(1 - \tau n_t^{AA}\right)(1 - \psi_t)\mathcal{N}_t, \tag{36}$$

taking into account households' time allocation towards child rearing.

5.3 Dynamics

Given the $K_0 > 0$ owned by the initial old generation, asset market clearing requires the usual condition

$$K_{t+1} = \left[\left\{\sigma_z^N(1 - \chi_t) + \sigma_z^A\chi_t\right\}(w_t + a_{1t})\,\psi_t + \sigma_z^A\,(w_t + a_{2t})\,(1 - \psi_t)\right]\mathcal{N}_t \tag{37}$$

that equates the supply of capital in $t + 1$ to aggregate wealth (savings) from t.

Definition 1. *A **dynamic equilibrium** of this economy consists of a sequence of allocations* $\{K_t, L_t, \mathcal{N}_t\}_{t=0}^{\infty}$*, prices* $\{w_t, r_t\}_{t=0}^{\infty}$*, fertility rates* $\{n_t^{NN}, n_t^{NA}, n_t^{AN}, n_t^{AA}\}_{t=0}^{\infty}$*, population shares* $\{\psi_t, \chi_t^{NA}, \chi_t\}_{t=0}^{\infty}$ *and the wealth threshold* $\hat{a}_t$ *such that*

i. Markets clear, that is, equations (36) *and* (37) *are satisfied,*

ii. Factor prices satisfy (31) *and* $r_t = \rho - 1$*,*

iii. $\hat{a}_t$ *is determined by* (34)*,* χ_t *by* (35)*,* *iv.* ψ_t *evolves according to* (32)*, and*

v. Aspiration choices are consistent with $\hat{a}_t$*,* ψ_t *and* χ_t

given $\psi_0 > 0$*,* $a_{10}, a_{20} > 0$*,* $K_0 > 0$*,* $\mathcal{N}_0^N > 0$ *and* $\mathcal{N}_0^A > 0$*.*

While the initial distribution of the population born into aspirational and non-aspirational households, ψ_0, is given, χ_0 is determined in equilibrium.

Balanced Growth Path

Distinguish economic regimes based on which technology is in use. In a *Malthusian regime*, applicable for all $t \in [0, T)$, production relies on the Malthusian technology, wage per worker is ω and the interest factor ρ. In a *Solovian regime*, only the Solovian technology is used during $t \in [T, \infty)$, wage per worker grows $w_t = \omega A_t = \omega A_0(1+g)^t$ over time while the interest factor is constant at ρ. Asymptotically only the Solovian technology is relevant.

Monotonicity of household wealth dynamics ensures that the balanced growth path (BGP) under the Solovian technology is unique and asymptotically stable as in the standard OLG model. In that BGP, output per worker (Y/N) and per unit of labor (Y/L), wage per unit of labor supply (w) and household wealth all grow at the constant rate g per generation while fertility rates (n^{NN}, n^{NA}, n^{AA}) are constant. If both aspirational and non-aspirational households coexist in the BGP, it is not necessary for them to have identical fertility rates, only that the proportion of each household type is constant.

Since bequests of aspirational and non-aspirational households are both proportional to the wage rate (see (19) and (24)), wage growth eventually pushes all families above δ/τ. After that, fertility rates for all households fall as the wage rate continues to grow. Exploiting the fact that $\lim_{t\to\infty} \delta/w_t = 0$ along the BGP, the four fertility rates asymptotically converge to

$$n^{NN*} = \sigma_n^N \left[\frac{1 + a_{1t}/w_t}{\tau}\right], \; n^{NA*} = \sigma_n^A \left[\frac{1 + a_{1t}/w_t}{\tau}\right], \; n^{AA*} = \sigma_n^A \left[\frac{1 + a_{2t}/w_t}{\tau}\right],$$

where $a_{jt}/w_t = \sigma_b^j \rho\tau/(1+g)$, $j \in \{N, A\}$, using (19), (24) and $w_{t+1}/w_t = 1+g$.

Proposition 3. *A* ***balanced growth path*** *(BGP) of this economy is a stationary equilibrium in which*

(i) Wage per worker increases at the constant rate g *while the interest rate is constant,*

(ii) Assets for non-aspirational (a_{1t}*) and aspirational (*a_{2t}*) agents and the wealth cutoff* $\hat{a}_t$ *growth at the constant rate* g*,*

(iii) Fertility rates are constant at $\{n^{NN*}, n^{NA*}, n^{AA*}\}$ and average fertility at $\bar{n}$,

(iv) Proportion of each cohort born into non-aspirational households is constant at ψ^*, and proportion born into non-aspirational households who choose to be aspirational at χ^*,

(v) Aggregate output and capital grows at the constant rate g and output per worker at the rate $(1+g)/(1+\bar{n})-1$ per generation.

5.4 Aspirations and Polarization

The steady-state proportion of young households born to non-aspirational parents $\psi_t = \psi_{t-1} = \psi^*$ solves

$$1 = n^{NN*}\left[\frac{1-\chi(\psi^*)}{\bar{n}(\psi^*)}\right] \tag{38}$$

from (32). Brute-force algebra yields the analytical solution

$$\psi^* = \frac{\Lambda}{2\kappa\rho\tau\Phi\left(\sigma_b^N - \sigma_{b2}\right)\left(\sigma_n^A\sigma_z^N - \sigma_n^N\sigma_z^A\right)}$$

where

$$\begin{aligned}\Lambda = &-(1+g+\rho\tau\sigma_b^N)\sigma_n^N + \sigma_n^A(1+g+\rho\tau\sigma_b^N - \kappa\Phi(1+g+\rho\tau\sigma_b^A)\sigma_z^N \\ &+\kappa\phi(1+g-\rho\tau+\sigma_b^N+2\rho\sigma_b^A)\sigma_n^N\sigma_z^A \\ &-\big[-4\kappa\rho\tau\Phi\left(\sigma_b^N-\sigma_b^A\right)\sigma_n^N(-1-g-\rho\tau\sigma_b^N+\kappa\Phi(1+g+\rho\tau\sigma_b^A)\sigma_z^A(\sigma_n^A\sigma_z^N-\sigma_n^N\sigma_z^A) \\ &\quad -(1+g+\rho\tau\sigma_b^N)\sigma_n^N+\sigma_n^A(1+g+\rho\tau\sigma_b^N-\kappa\Phi(1+g+\rho\tau\sigma_b^A)\sigma_z^N \\ &\quad +\kappa\Phi(1+g-\rho\tau\sigma_b^N+2\rho\tau\sigma_b^A)\sigma_n^N\sigma_z^A\big]^{1/2}.\end{aligned}$$

As expected ψ^* is monotonically increasing in λ.

Similar to section 3, we are interested in understanding whether or not aspirations is evolutionarily stable under endogenous fertility. As we showed in section 3, the preference externality from aspirations formation introduces a tendency for polarization of wealth and aspirations. Because we are looking at stationary equilibria where the fertility of the poor (non-aspirational households) exceeds that of the rich (aspirational households), the faster replication rate of the poor, *ceteris paribus*, tends to lower $\bar{z}$ and make aspirations more attainable that before.

Does that mean everyone becomes aspirational in the long run? This depends on how strongly people respond to their aspirations as proxied by the parameters (κ, λ).

Proposition 4. *The BGP can be one of four types distinguished by the share of aspirational households and degree of polarization.*

(i) For $\psi^* = 1$, all households are non-aspirational and there is no inequality of wealth or lifetime utility,

(ii) For $\psi^* = 0$, all households are aspirational and there is no inequality of wealth or lifetime utility,

(iii) For $\psi^* < 1$, $\chi^* = 0$, poorer non-aspirational households co-exist with richer aspirational ones due to which there is persistent inequality of wealth and lifetime utility between the two groups,

(iv) For $\psi^* < 1$, $\chi^* > 0$, three types of households – poor, middle-class and rich – co-exist, the latter two being aspirational, the first two differing in wealth but not lifetime utility, and there is persistent inequality of wealth and lifetime utility between the non-aspirational poor and aspirational rich.

Proof. See Appendix D. □

Thus, while aspirations can be a dynamically stable population trait, it is not necessarily evolutionarily stable because, for cases (i), (iii) and (iv), the entire population does not become aspirational. The proof shows that $\psi^* = 0$ occurs either for low values of κ (aspirations level relatively attainable by poorer households), or, if κ is high, for low values of λ (modest behavioral gap between aspirational and non-aspirational households). The similarity in the role played by these parameters should not be surprising as κ and λ are complementary inputs in aspirations formation. While κ dictates how high the aspirations level is set, λ determines how much one responds to that aspirations. For the asset-poor what is relevant is the cost of narrowing their aspirations gap and here κ and λ work similarly.

Of course, in our model the aspirational benchmark $\kappa\bar{z}$ is exogenous to households while λ is (effectively) chosen. We leave it to future work how the choice of κ itself may be shaped by socio-economic factors. For example, exposure to and information about the economic successes of professional peers and role models can affect the benchmark cross-sectionally and over time.[12] In principle, the same factors can heighten the responsiveness

[12] In Mookherjee *et al.* (2010), parents aspire with respect to their children's income and, in choosing where to locate, take into account the effect of neighborhood characteristics on their children's education. Location choice, therefore, directly affects aspirations formation. The authors show that segregated neighborhoods differing in average skill levels can arise in steady state. The choice of the aspirational benchmark, in other words, can lead to polarization much as the choice to be aspirational does.

to aspirations. But the choice to be aspirational depends fundamentally on one's perception of their ability to meet those aspirations. This is where institutions matter. Career paths that lead to exceptional economic success may not be widely available no matter how high one wants to reach because of malfunctioning credit markets, institutionalized discrimination, or insecure property rights. Rational individuals may, in this world, view the pursuit of aspirations as futile. Similarly, social and cultural factors that alter perceptions of the role of effort versus luck in determining lifetime income, as in Alesina and Angeletos (2005), can determine how strongly people aspire.

5.5 A Numerical Example

We wrap up the discussion using a numerical example. Since the parameter space is large, the values selected in Table 1 are meant to be somewhat realistic.

$\rho = 1.05^{25}$	$\beta = 0.96^{25}$
$\omega = 7.5$	$\gamma = 0.69$
$A_0 = 0.001$	$\tau = 0.15$
$g = 1.02^{25} - 1$	$\delta = 0.1$
$\psi_0 = 0.5$	$\kappa = 1$

Table 1: Parameter Values

As baseline, we set $\lambda = 1$ and pick $\theta = 0.648$ so that fertility is at replacement in the BGP, $\bar{n}^* = 1$.[13] The values of ρ and g are picked to ensure an annual real interest rate of 5% and BGP annual growth rate of 2% for output per worker. The subjective discount rate β is standard while A_0, ψ_0 and κ are arbitrarily picked since they are scaling parameters. The time cost of child-rearing, τ, is set according to the literature (for example, Aksan and Chakraborty, 2014) while the resource cost δ, a scale parameter, is chosen to generate a positive fertility-income relationship in the Malthusian regime.

The Long Run

We initialize the economy with $a_{10} = 0$, $a_{20} = 1$, $\psi_0 = 0.5$ and $K_0 = 600$, and choose $A_0 = 0.001$ so that the Malthusian technology is more productive early on. In the long run, only the Solovian technology is use, there is secular income growth from exogenous labor

[13] Since our interest lies in the formation and evolution of aspirations, we also entertain other values of $\lambda \in (0, 1)$. In each case, $\bar{n}^* = 1$ is maintained by varying θ, e.g. $\theta = 0.645$ when $\lambda = 0.5$.

productivity growth, and the relationship between between fertility and labor income is negative for all households. Moreover, in line with Proposition 4-(iv), there is persistent inequality in wealth and lifetime utility between the aspirational rich and non-aspirational poor.

Consider how λ, the parameter that governs aspirations formation, affects this steady-state. The higher is λ, the larger the gap in wealth accumulation and fertility behavior – first panel in Figure 5 – between the aspirational and non-aspiration households. Higher values of λ increase the difference in wealth (a) between the two groups, raising $\bar{z}$. They also create a higher fertility gap between the two groups which tends to lower $\bar{z}$ as the poor have more children. Evidently, the first effect strongly dominates (second panel) as the proportion of non-aspirational households increases with λ. Predictably, lifetime indirect utility gaps between aspirational (V^A) and non-aspirational households (V^N) is higher too (third panel) for higher values of λ.

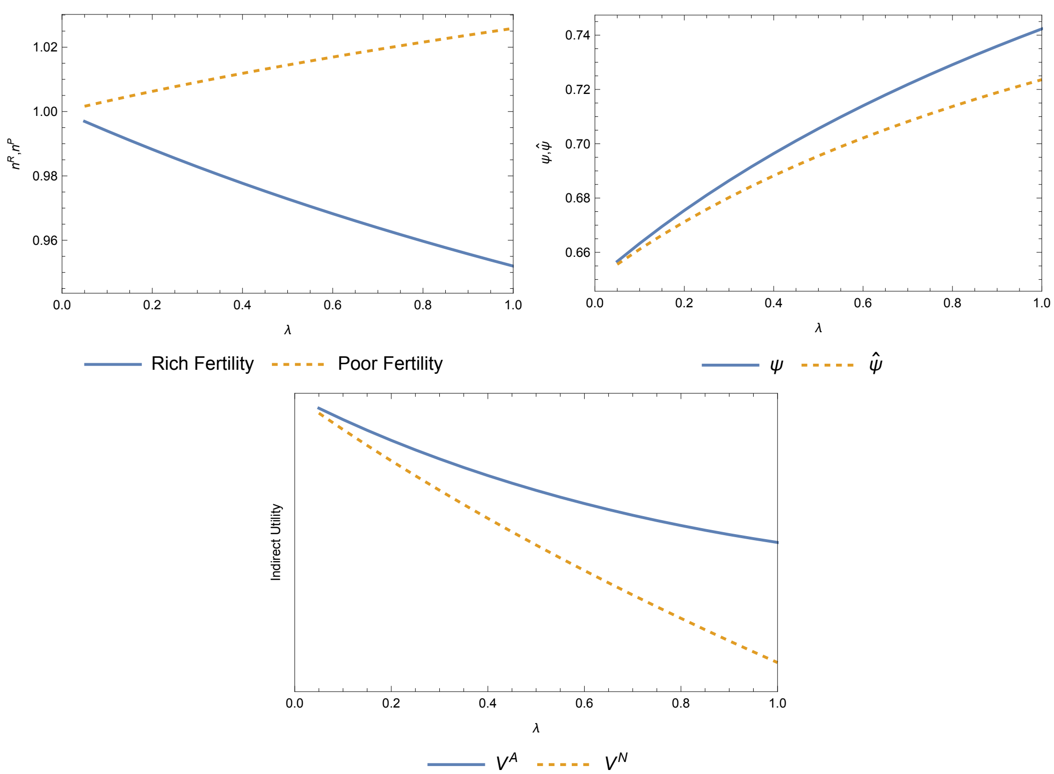

Figure 5: Effect of λ on the steady state

Transition to the Long Run

Turn next to transitional dynamics. As can be seen in Figure 6, in the Malthusian regime, output per capita is roughly constant while aggregate output shows some growth due to population increase and capital accumulation. The switch to the Solovian technology at $T = 14$ produces quick gains in output per capita.

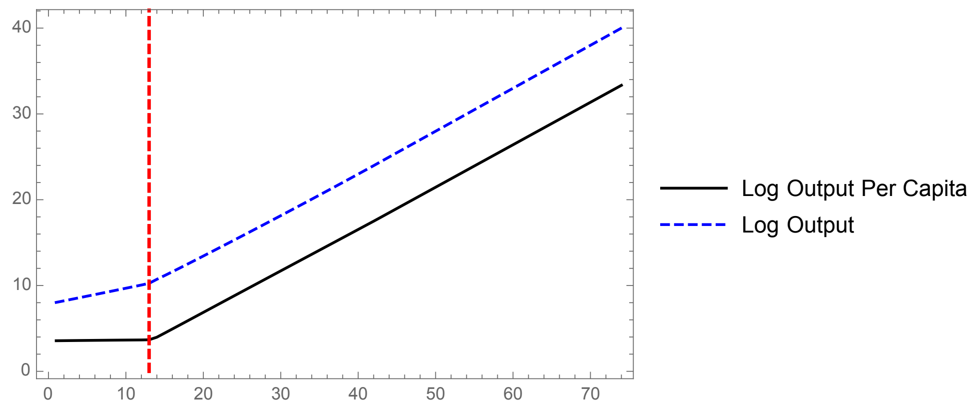

Figure 6: Path of Output and Output per capita

The evolution of aspirations during transition to the steady state depends on non-monotonic demographic change.[14] Take the fertility ratio between the poorest non-aspirational and wealthiest aspirational households. In the Malthusian regime, this ratio is

$$\left(\frac{n_t^{NN}}{n_t^{AA}}\right)_M = (1+\lambda_n)\left[\frac{\omega(1+\rho\tau\sigma_b^N)+\rho\delta\sigma_b^N}{\omega(1+\rho\tau\sigma_b^A)+\rho\delta\sigma_b^A}\right] \equiv \eta_M, \tag{39}$$

where $\lambda_n \equiv \lambda/(1+\beta+\gamma\theta)$. Clearly η_M is increasing in wage per worker ω, and decreasing in the resource (δ) and time costs (τ) of child-rearing. In the Solovian regime,

$$\left(\frac{n_t^{NN}}{n_t^{AA}}\right)_S = \frac{\sigma_n^N}{\sigma_n^A}\left[\frac{1+g+\sigma_b^N\rho\tau}{1+g+\sigma_b^A\rho\tau}\right] \equiv \eta_S \tag{40}$$

is increasing in τ but independent of the wage rate per effective worker ω. Since $\sigma_b^A > \sigma_b^N$, we have $\eta_M < \eta_S$. This is due to the fertility advantage of wealth in the Malthusian regime in contrast to the Solovian regime. The parameter values of Table 1 imply that $\eta_M < 1 < \eta_S$. Figure 7 shows the time-path of the fertility ratio η (left panel) and fertility rates n_t^{NN}, n_t^{AA} (right panel). Note particularly how quickly both rich and poor fertility respond to the substitution effect.

The proportion ψ of households born into non-aspirational families behaves non-monotonically over time as shown in Figure 8. In the Malthusian regime, the fraction of the population that is born into aspirational households, $1-\psi_t$, steadily increases.

[14] In the model, this non-monotonic change is driven by the switch from less to more productive technologies. This approach has a long lineage – see Hansen and Prescott (2002) and Galor (2005) – though the timing of technology and demographic switch need not coincide.

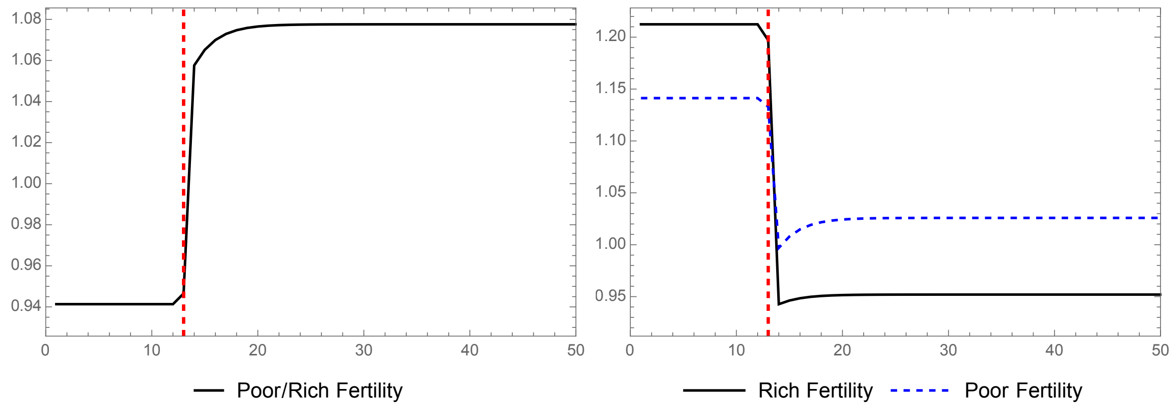

Figure 7: Fertility Rates

After T, the pattern reverses and the population steadily gets less aspirational.

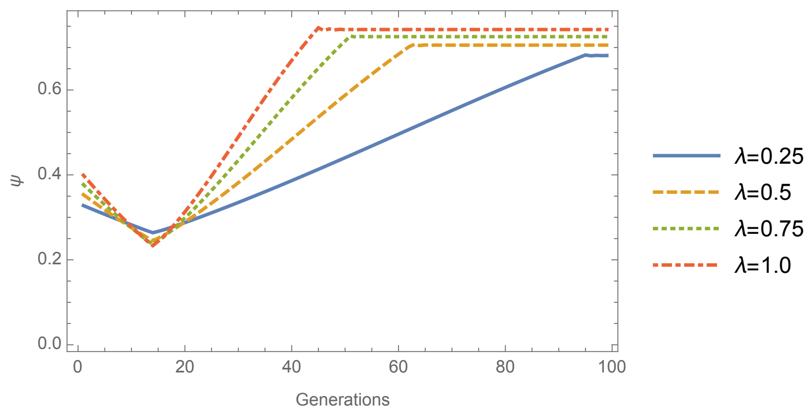

Figure 8: Proportion of Non-aspirational Agents, ψ_t

Of course, we also need to consider χ, the proportion of children born into non-aspirational households who become aspirational. From Figure 9, this proportion is trivial (left panel) for much of the transition and, even in the BGP, remains small.

Finally consider the evolution of inequality in lifetime utilities between the aspirational rich (V^A) and non-aspirational poor (V^N). Figure 10 shows how this behaves non-monotonically, mimicking the non-monotonic path of aspirations formation in the economy.

Other Outcomes

Though Table 1 implies $\eta_M < 1 < \eta_S$, it is not the only theoretical possibility. Two other scenarios can occur for alternative parameter values: $\eta_M < 1 < \eta_S$ or $1 < \eta_M < \eta_S$.

The first one, $\eta_M < \eta_S < 1$, requires much higher values of (ρ, τ) than used in Table 1. More importantly, we can ignore this scenario as it is empirically irrelevant; the fertility

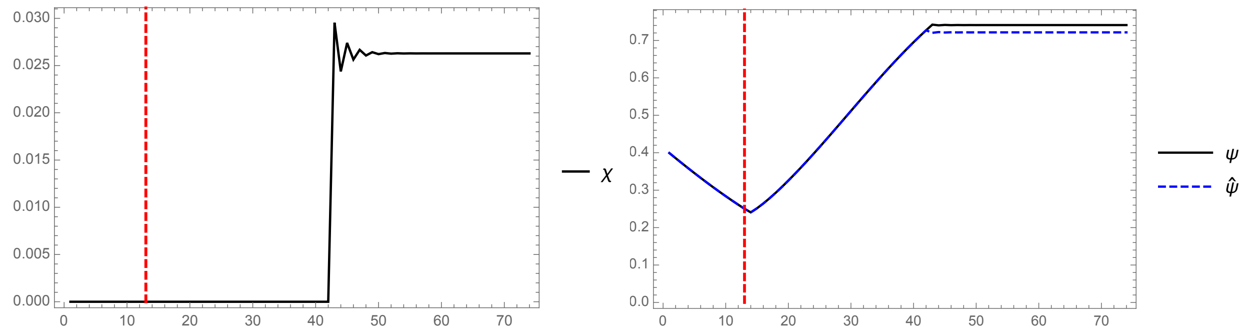

Figure 9: Path of χ_t, ψ_t and $\hat{\psi}_t$

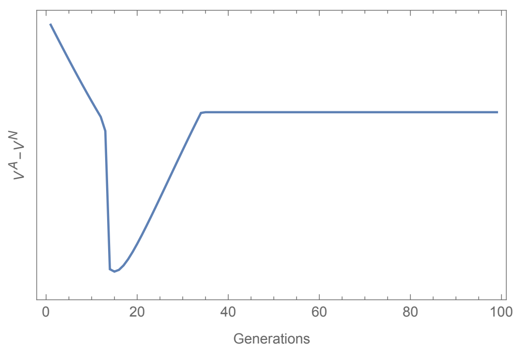

Figure 10: Inequality of Lifetime Utilities

rate of non-aspirational poor households is lower than that of the aspirational rich agents in the long run.

We conclude with a brief description of the other scenario, $1 < \eta_M < \eta_S$, where non-aspirational households have a higher fertility rate than aspirational households in both regimes. By reproducing faster, the non-aspirational poor make it feasible for aspirations to be attained by a large share of the population. The stationary distribution of wealth is, however, bimodal with families converging to high wealth and aspirations or low wealth and no aspirations. Figure 11 provides an illustration using $\tau = 0.05$, $\theta = 0.85$, and $\rho = 1.01^{25}$, other values being the same as in Table 1. The rental rate ρ matters because bequests are funded out of lifetime savings; a lower (expected) return on savings weakens the quantity-quality tradeoff ensuring that fertility levels are high. The corresponding proportion of switchers in the long run is a high 16%.

6 Conclusion

We constructed an overlapping generations model of endogenous preference formation. Forward-looking households optimally choose whether or not to be aspirational, a

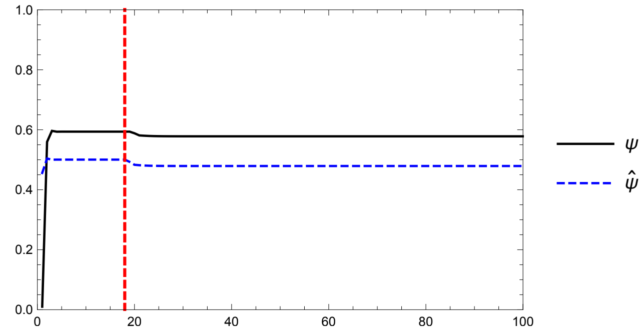

Figure 11: Path of ψ and $\hat{\psi}$ when $1 < \eta_M < \eta_S$

decision that turns on inherited wealth. This creates a tendency toward persistent inequality without the conventional margins such as credit frictions and exogenous productivity used in much of the literature. Fertility choice intensifies this tendency if the rich have more children, dilutes it if they have fewer. We illustrate conditions under which unconditional convergence does not occur and aspirations-formation generates long-run inequality in wealth and fertility.

Our framework takes the aspirational benchmark itself to be exogenous. What if people could choose what to aspire to or how strongly to aspire? On the one hand, if the poor could set a lower hurdle for themselves or respond weakly, they may well choose to be aspirational more often than not. Differential aspirational behavior between the rich and poor whether in terms of the benchmark or responsiveness to the benchmark would, however, work the same way as the choice whether or not to aspire. That being said, entertaining these possibilities opens the door for factors such as occupation, location, social networks and culture to matter more for aspirations than just the wealth distribution. It can also help us understand how the incentive to aspire changes with the rise of material well-being.

Appendix A: Aspirations with respect to bequest

We show here that the basic framework is robust to aspirations with respect to children's well-being. Suppose the aspirations function is $\alpha_t = \ln[\bar{b}_{t+1}/b_{t+1}]$ and the agent maximizes

$$U_t = \ln c_{1t} + \beta \ln c_{2t+1} + \gamma(1-\theta)\ln b_{t+1} - \lambda \mathcal{I}_t \alpha_t$$

subject to the budget constraints

$$c_{1t} + z_t = w_t + a_t,\ c_{2t+1} + b_{t+1} = R_{t+1} z_t.$$

An agent that opts out ($\mathcal{I}_t = 0$) chooses

$$\begin{aligned} c_{2t+1} &= \frac{\beta}{1+\beta+\gamma(1-\theta)}(w_t + a_t) &&\equiv \tilde{\mu}^N_{c2}(w_t + a_t) \\ z_t &= \frac{\beta+\gamma(1-\theta)}{1+\beta+\gamma(1-\theta)} R_{t+1}(w_t + a_t) &&\equiv \tilde{\mu}^N_z R_{t+1}(w_t + a_t) \\ b_{t+1} &= \frac{\gamma(1-\theta)}{1+\beta+\gamma(1-\theta)} R_{t+1}(w_t + a_t) &&\equiv \tilde{\mu}^N_b R_{t+1}(w_t + a_t) \end{aligned}$$

while an aspirational agent ($\mathcal{I}_t = 1$) chooses

$$\begin{aligned} c_{1t} &= \frac{1}{1+\beta+\gamma(1-\theta)+\lambda}(w_t + a_t) &&\equiv \tilde{\mu}^A_{c1}(w_t + a_t) \\ c_{2t+1} &= \frac{\beta}{1+\beta+\gamma(1-\theta)+\lambda}(w_t + a_t) &&\equiv \tilde{\mu}^A_{c2}(w_t + a_t) \\ z_t &= \frac{\beta+\gamma(1-\theta)+\lambda}{1+\beta+\gamma(1-\theta)+\lambda} R_{t+1}(w_t + a_t) &&\equiv \tilde{\mu}^A_z R_{t+1}(w_t + a_t) \\ b_{t+1} &= \frac{\gamma(1-\theta)+\lambda}{1+\beta+\gamma(1-\theta)+\lambda} R_{t+1}(w_t + a_t) &&\equiv \tilde{\mu}^A_b R_{t+1}(w_t + a_t) \end{aligned}$$

Then comparing indirect utilities from participation versus opting out, a household decides to aspire if

$$a_t \geq \left[\frac{\mu^N_{c1}(\mu^N_{c2})^\beta(\mu^N_b)^{\gamma(1-\theta)}}{\mu^A_{c1}(\mu^A_{c2})^\beta(\mu^A_b)^{\gamma(1-\theta)+\lambda}}\right]\left(\frac{\bar{b}_{t+1}}{R_{t+1}}\right) - w_t \equiv \hat{a}_t$$

similar to before. Since bequests are an increasing function of household income, under appropriate conditions, aspirations formation can lead to persistent inequality similar to the model in the paper.

Appendix B: Proof of Proposition 2

Two locally stable steady-states of (14) require that $a_1^* < \hat{a} < a_2^*$. Since steady-state aggregate saving is

$$\bar{z} = \psi z_1 + (1-\psi) z_2 = \psi \mu_{1z}(\omega + a_1^*) + (1-\psi)\mu_{2z}(\omega + a_2^*),$$

from (14) this requires that

$$\xi_2 > \left[\frac{1 - \Omega\kappa\psi\mu_{1z}}{\Omega\kappa(1-\psi)\mu_{2z}}\right]\xi_1$$

and

$$\xi_2 > \left[\frac{\Omega\kappa\psi\mu_{1z}}{1 - \Omega\kappa(1-\psi)\mu_{2z}}\right]\xi_1.$$

The parametric condition (16) follows.

Appendix C: χ^{NA} and χ^{AN}

Allowing for the possibility that children of aspirational households may choose to be non-aspirational requires the exposition of section 5.3 to be modified. For the fertility rates we have

$$n_t^{NN} = \sigma_n^N \left[\frac{w_t + a_{1t}}{\tau w_t + \delta}\right], \quad n_t^{NA} = \sigma_n^A \left[\frac{w_t + a_{1t}}{\tau w_t + \delta}\right], \quad n_t^{AN} = \sigma_n^N \left[\frac{w_t + a_{2t}}{\tau w_t + \delta}\right], \quad n_t^{AA} = \sigma_n^A \left[\frac{w_t + a_{2t}}{\tau w_t + \delta}\right],$$

using which the aspirations groups are

$$\mathcal{N}_{t+1}^N = (1-\chi_t^{NA}) n_t^{NN}\mathcal{N}_t^N + \chi_t^{AN} n_t^{AN}\mathcal{N}_t^A, \quad \mathcal{N}_{t+1}^A = \chi_t^{NA} n_t^{NA}\mathcal{N}_t^N + (1-\chi_t^{AN}) n_t^{AA}\mathcal{N}_t^A.$$

Now $\psi_t \equiv \mathcal{N}_t^N/\mathcal{N}_t$ evolves according to

$$\psi_{t+1} = \frac{(1-\chi_t^{NA}) n_t^{NN}\psi_t + \chi_t^{AN} n_t^{AN}(1-\psi_t)}{\bar{n}_t}$$

where

$$\bar{n}_t \equiv \frac{\mathcal{N}_{t+1}}{\mathcal{N}_t} = \left[\left(1-\chi_t^{NA}\right) n_t^{NN} + \chi_t^{NA} n_t^{NA}\right]\psi_t + \left[\chi_t^{AN} n_t^{AN} + (1-\chi_t^{AN}) n_t^{AA}\right](1-\psi_t).$$

The χ^{ij}'s are determined by the wealth threshold $\hat{a}$

$$\hat{a}_t = \kappa\Phi\left[\left\{(1-\chi_t^{NA}) z_t^{NN} + \chi_t^{NA} z_t^{NA}\right\}\psi_t + \left\{\chi_t^{AN} z_t^{AN} + (1-\chi_t^{AN}) z_t^{AA}\right\}(1-\psi_t)\right] - w_t.$$

Using this, as before, the equilibrium χ_t^{NA} is specified as

$$\chi_t^{NA} = \begin{cases} 0, & \text{if } \hat{\chi}_t^{NA} < 0 \\ \hat{\chi}_t^{NA}, & \text{if } 0 < \hat{\chi}_t^{NA} < 1 \\ 1, & \text{if } \hat{\chi}_t^{NA} > 1. \end{cases}$$

where

$$\hat{\chi}_t^{NA} = \frac{\kappa\Phi\psi_t\left[\sigma_z^N(w_t + a_{1t}) - \sigma_z^A(w_t + a_{2t})\right] + \kappa\Phi\sigma_z^A(w_t + a_{2t}) - (w_t + a_{1t})}{\kappa\Phi\psi_t(w_t + a_{1t})(\sigma_z^N - \sigma_z^A)}.$$

Likewise,

$$\chi_t^{AN} = \begin{cases} 0, & \text{if } \hat{\chi}_t^{AN} < 0 \\ \hat{\chi}_t^{AN}, & \text{if } 0 < \hat{\chi}_t^{AN} < 1 \\ 1, & \text{if } \hat{\chi}_t^{AN} > 1. \end{cases}$$

where

$$\hat{\chi}_t^{AN} = \frac{\kappa\Phi\psi_t\left[\sigma_z^N(w_t + a_{1t}) - \sigma_z^A(w_t + a_{2t})\right] + \kappa\Phi\sigma_z^A(w_t + a_{2t}) - (w_t + a_{2t})}{\kappa\Phi(1 - \psi_t)(w_t + a_{2t})(\sigma_z^A - \sigma_z^N)}.$$

$\chi^{ij} > 0$ implies that $\chi^{ji} = 0$ for $j \neq i$. Therefore, two types of BGP can occur: (I) $\chi^{NA*} > 0$, $\chi^{AN*} = 0$, and (II) $\chi^{AN*} > 0$, $\chi^{NA*} = 0$. A unique steady-state ψ^* exists for (I) which occurs when $n^{NN*} > n^{AA*}$, that is, poor fertility exceeds rich fertility in the long run. Case (II), on the other hand, requires $n^{NN*} < n^{AA*}$. Rich fertility exceeding poor fertility is inconsistent with post-demographic transition societies. But it may ((temporarily) apply to a Malthusian pre-demographic transition society that has settled into a pseudo steady state before the Solovian technology take-off at T.

Appendix D: Proof of Proposition 4

(i) Everyone is non-aspirational $\psi^* = 1$

We show that $\psi^* = 1$ is a feasible stationary equilibrium.

Proof. First we show that this is true if $\kappa > 1/(\Phi\sigma_z^N)$. Suppose that everyone in the economy is non-aspirational and has asset holdings a_1. The cut-off asset level for aspirations is:

$$\hat{a} = \kappa\Phi\sigma_z^N(w + a_1) - w$$

Everyone in the economy will continue to be non-aspirational if $\hat{a} > a_1$, that is, $\kappa\Phi\sigma_z^N(w + a_1) - w > a_1$ which can be rearranged to:

$$(\kappa\Phi\sigma_z^N - 1)w > a_1(1 - \kappa\Phi\sigma_z^N).$$

This is unambiguously true if $\kappa > 1/(\Phi\sigma_z^N)$.

Next suppose this were not the case, that is, $\kappa \leq 1/(\Phi\sigma_z^N)$. We show that $\psi^* = 1$ is feasible for a high enough value of λ. From above, we know that everyone being non-aspirational in the long-run requires $\kappa\Phi\sigma_z^N > 1$. It is straightforward to show:

$$\kappa\Phi\sigma_z^N = \kappa\left(\frac{\beta+\gamma(1-\theta)}{1+\beta+\gamma\theta}\right)^{1+\beta}\left(\frac{1+\beta+\gamma\theta+\lambda}{\beta+\gamma(1-\theta)+\lambda}\right)^{\beta+\lambda} \times\left(1+\frac{\lambda}{\beta+\gamma(1-\theta)}\right)^{\gamma(\theta-1)}\left(\frac{\lambda}{\beta+\gamma\theta+1}+1\right)^{1+\gamma\theta}$$

When $\lambda = 0$,

$$\kappa\Phi\sigma_z^N = \kappa\left(\frac{\beta+\gamma(1-\theta)}{1+\beta+\gamma\theta}\right) < 1$$

and $\lim_{\lambda\to\infty}\kappa\Phi\sigma_z^N \to \infty$. We also have that

$$\frac{\partial\kappa\Phi\sigma_z^N}{\partial\lambda} = \frac{\kappa}{\beta+\gamma\theta+1}\left[\left(\frac{\beta+\gamma(1-\theta)}{\beta+\gamma\theta+1}\right)^{\beta+1} \times(\beta+\gamma\theta+\lambda+1)\left(\frac{\beta+\gamma\theta+\lambda+1}{\beta+\gamma(1-\theta)+\lambda}\right)^{\beta+\lambda}\left(\frac{\lambda}{\beta+\gamma(1-\theta)}+1\right)^{\gamma(\theta-1)} \times\left(\frac{\lambda}{\beta+\gamma\theta+1}+1\right)^{\gamma\theta}\ln\left(\frac{\beta+\gamma\theta+\lambda+1}{\beta+\gamma(1-\theta)+\lambda}\right)\right]$$

This is unambiguously positive if $\ln\left(\frac{\beta+\gamma\theta+\lambda+1}{\beta+\gamma(1-\theta)+\lambda}\right) > 0$, that is, if $\frac{\beta+\gamma\theta+\lambda+1}{\beta+\gamma(1-\theta)+\lambda} > 1$, which is always true.

Hence, by the Intermediate Value Theorem, there exists a $\bar{\lambda}_1$ for which

$$\text{If } \lambda \leq \bar{\lambda}_1 \text{ then } \kappa\Phi\sigma_z^N < 1$$
$$\text{If } \lambda > \bar{\lambda}_1 \text{ then } \kappa\Phi\sigma_z^N > 1$$

In other words, for $\kappa \leq 1/(\Phi\sigma_z^N)$, $\psi^* = 1$ if $\lambda > \bar{\lambda}_1$. □

(ii) Everyone is aspirational, $\psi^* = 0$

Proof. Suppose that everyone in the economy is aspirational and has asset holdings a_2. The cut-off asset level for aspirations is:

$$\hat{a} = \kappa\Phi\sigma_z^A(w+a_2) - w$$

$\psi^* = 0$ is possible iff $\hat{a} < a_2$, that is,

$$\kappa\Phi\sigma_z^A(w+a_2) - w < a_2 \Leftrightarrow (\kappa\Phi\sigma_z^A - 1)w < a_2(1-\kappa\Phi\sigma_z^A)$$

which is unambiguously true if $\kappa < 1/(\Phi\sigma_z^A)$.

Now suppose this is not the case, that is, $\kappa \geq 1/(\Phi\sigma_z^A)$. Again, from above, we know that everyone being aspirational in the long run requires $\kappa\Phi\sigma_z^A < 1$. We have

$$\kappa\Phi\sigma_z^A = \kappa(\beta+\gamma(1-\theta))^\beta \left(\frac{1}{\beta+\gamma\theta+1}\right)^{\beta+\gamma\theta+1} \times (\beta+\gamma(1-\theta)+\lambda)^{-\beta-\lambda+1}(\beta+\gamma\theta+\lambda+1)^{\beta+\gamma\theta+\lambda}\left(\frac{\lambda}{\beta+\gamma(1-\theta)}+1\right)^{-\gamma(1-\theta)}$$

When $\lambda = 0$, then

$$\kappa\Phi\sigma_z^A = \kappa\left(\frac{\beta+\gamma(1-\theta)}{1+\beta+\gamma\theta}\right) < 1$$

and $\lim_{\lambda\to\infty}\kappa\Phi\sigma_z^A \to \infty$. Moreover,

$$\frac{\partial\kappa\Phi\sigma_z^A}{\partial\lambda} = (\beta+\gamma(1-\theta))^\beta\left(\frac{1}{\beta+\gamma\theta+1}\right)^{\beta+\gamma\theta+1}(\beta+\gamma(1-\theta)+\lambda)^{-\beta-\lambda} \times (\beta+\gamma\theta+\lambda+1)^{\beta+\gamma\theta+\lambda-1}\left(\frac{\lambda}{\beta+\gamma(1-\theta)}+1\right)^{\gamma(\theta-1)} \times \left(1+\gamma(2\theta-1)-(\beta+\gamma(1-\theta)+\lambda)(\beta+\gamma\theta+\lambda+1)\ln\left(\frac{\beta+\gamma(1-\theta)+\lambda}{\beta+\gamma\theta+\lambda+1}\right)\right)$$

which is unambiguously positive if $\ln\left(\frac{\beta+\gamma(1-\theta)+\lambda}{\beta+\gamma\theta+\lambda+1}\right) < 0$ which is true because $\frac{\beta+\gamma(1-\theta)+\lambda}{\beta+\gamma\theta+\lambda+1} < 1$. Hence, there exists a $\bar{\lambda}_2$ for which

$$\text{If } \lambda \le \bar{\lambda}_2 \text{ then } \kappa\Phi\sigma_z^A < 1$$
$$\text{If } \lambda > \bar{\lambda}_2 \text{ then } \kappa\Phi\sigma_z^A > 1$$

We conclude that when $\kappa > 1/(\Phi\sigma_z^A)$, $\psi^* = 0$ is a long-run equilibrium as long as $\lambda < \bar{\lambda}_2$, where $\bar{\lambda}_2$ is defined as $\kappa\Phi(\bar{\lambda}_2)\sigma_z^A(\bar{\lambda}_2) = 1$. □

(iii) Bi-modal distribution, $\psi^* < 1, \chi^* = 0$

Proof. If $\psi^* < 1$, that implies that there is inherited wealth inequality such that: $a_1^* < \hat{a}_t < a_2^*$. Equation (30) tells us that $V^N(a_1^*) < V^A(a_2^*)$. Therefore the two types of households differ in lifetime utility. From equations (17) and (22), aspirational households will save more than the non-aspirational households, $a_2^* > a_1^*$. Therefore, they differ in lifetime wealth. □

(iv) Tri-modal distribution $\psi^* < 1, \chi^* > 0$

Proof. $\psi^* < 1$ and $\chi^* > 0$ imply $a_1^* = \hat{a}_t < a_2^*$. Since $a_1^* = \hat{a}_t$, we have $V^N(a_1^*) = V^A(a_1^*)$. Children of non-aspirational households who switch from their parent's type have the same inherited wealth level as children of non-aspirational households who do not. Therefore, they must be indifferent between becoming aspirational versus not: lifetime utility of the middle class (switchers) and the poor (non-switchers) is the same. However, since the switchers are now aspirational, they have a higher savings propensity and the same inherited assets as the non-switchers, which implies greater overall savings and higher lifetime wealth. □

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