

Understanding the Informal Sector: Do Informal and Formal Firms Compete?

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Abstract

The literature on informality has struggled to characterize the role that informal firms play in the macroeconomy. We seek to address a central component of this confusion: do formal and informal sector firms compete? We highlight survey evidence that strongly suggests competition between the formal and informal sector. Motivated by this finding, we construct a theoretical model that resolves lingering conflicts in the literature. The model predicts different patterns of competition across sectors of the economy. We are able to show that within an industry informal firms tend to be smaller and less productive than formal firms; however, in aggregate terms we are able to qualitatively match the evidence that shows an overlap between the productivity distributions of the formal and informal sectors.

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1 Introduction

Informality and economic development have always been inexorably linked. So much so that many theories of economic development specifically incorporate informality. Despite this fundamental role in the field of development, there has been relatively little consensus on how to characterize the role that informal firms play in the macroeconomy. This lack of consensus undoubtedly spans from the difficulties of accurately measuring informal economies. Past studies have relied on detailed anecdotes, country studies, second-hand survey data, and a plethora of correlated aggregate measures, all of which have clear drawbacks.

Particularly problematic for understanding informality is the current divide between theory and empirical evidence. Rauch (1991) develops the canonical model for explaining which entrepreneurs will decide to operate informal firms. A central result of the model is a strict size and productivity dualism. The smallest formal sector firm is larger than the largest informal sector firm. A similar strict cut-off exists in terms of firm productivity: the least productive formal sector firm is still more productive than the most productive informal sector firm. However, several papers have documented overlapping productivity distributions between the formal and informal sector. The clearest documentation of this overlap is illustrated in Nataraj (2011) for the case of India.¹

In this paper, we attempt to bridge this divide and develop a model to better understand how and when formal and informal firms compete. A concrete understanding of how firms from different sectors compete is of first-order importance to create and implement policy in developing countries. It is also a fundamental, if

¹ Further insights can be found by comparing the distribution of plant sizes in Hsieh and Klenow (2009) relative to Hsieh and Klenow (2010).

overlooked, element of many macroeconomic models that incorporate informality. We begin by highlighting empirical data that suggests a large degree of competition between the formal and informal sectors.

Motivated by this finding we construct a multi-industry version of the standard Melitz (2003) model. In addition to making decisions about entry, production, and pricing, firms must decide whether to operate formally or informally. This framework allows us to make clear predictions about which industries informal firms tend to operate in, how they compare to formal firms, and whether the two types of firms compete. Within a given industry, the model maintains the strict size-dualism theorized by Rauch (1991). However, across industries the strict size-dualism no longer holds, allowing aggregate distributions of productivity and firm size to qualitatively match the patterns in the data. Higher productivity firms face less competition, as measured by a smaller decline in profits from informal competition. At an industry level, this reflects higher productivity industries having less competition between formal and informal firms.

The broader literature on informality provides a helpful context for this work. It generally falls into three categories, although there are certainly overlaps between them. On one end of the spectrum, De Soto (1989, 2000) argues the informal sector exists due to restrictive institutional constraints. In his view, institutional reforms would unleash the creativity and entrepreneurial spirit of the informal sector. He writes “[i]n all cases, however, it is not the characteristics of the enterprise but rather those of the institutional framework that determine the boundaries of informality (1989, xxii).” In a sense, he argues that the rise of informality is a best response to inept institutions. Other authors argue that informality is simply a profit-driven decision, and informal entrepreneurs operate informally to gain a competitive advance. For instance, Farrell (2004) argues that informal firms operate outside of the formal

sector for the sole purpose of avoiding regulations and taxation. This gives them a considerable cost advantage relative to formal sector firms and cuts into the profits of firms in the formal sector. This view is similar to that of Levy (2008).

Between these two points of view, there has emerged a consensus of sorts. It centers on Lewis (1954) which posits a dual view of the economy. His work suggests that informality exists as a stage in the process of development, and, importantly, dissipates as countries develop. It argues that there is a strict separation between formal and informal firms in what or how they produce or who they serve. This branch of the literature includes seminal theoretical contributions such as Harris and Todaro (1970) and Rauch (1991). These theoretical models have been largely substantiated by empirical investigations done by La Porta and Shleifer (2008, 2014).

La Porta and Shleifer (2008), in particular, documents important facets of informality that are supportive of the dual view. For instance, they find that 91% of formal, or registered, firms began that way. This is similar to an observation in Nataraj (2011) that very few informal Indian firms ever switch sectors. In addition to this seemingly impermeable barrier between sectors, there is evidence to suggest informal firms may produce different goods. La Porta and Shleifer (2008) find considerable differences in value-added between the formal and informal sector, suggesting that formal firms produce higher value goods than informal firms. La Porta and Shleifer (2014) add more evidence for the dual view with updated findings such as only 2% of informal firms sell to larger firms and 14% of all firms do so. This difference speaks to the assertion that informal firms are largely divorced from the economic markets served by formal sector firms. In totality, these works provide a convincing argument for the dual view of informality.

This work contributes to the literature in two distinct and important ways. First, it resolves the conflicting theoretical and empirical results that shows strict size

dualism and overlapping productivity distributions. In doing so, we argue that our model *strengthens* the dual view of informality by reconciling it with existing evidence on size and productivity distributions in developing countries like India. Additionally, the data that we use to motivate our research is consistent with a developmental process where informality generally dissipates with economic development.

Our second contribution is to document and illustrate the degree of competition within industries. Our results encourage a more nuanced view of competition between formal and informal firms. Supporters of the dual view often see formal and informal firms occupying very different parts of the economy. While this may be true in some industries, other industries see a large degree of competition between both types of firms. Available survey data from the World Bank, as well as results from our theoretical model, underscore a level of competition between formal and informal firms that is difficult to reconcile with some interpretations of the dual view.

In summary, this paper reaches three important conclusions that help frame how we should view informality. First, within a given industry, the largest, most productive firms will be formal. Second, across industries, there will be some informal firms that are more productive than formal sector firms. Finally, informal and formal firms compete, with the degree of competition, as measured by a loss in profits to formal firms, decreasing as a function of formal firms' productivity.

This paper is divided into five sections, with this introduction constituting Section 1. Section 2 presents our empirical motivation. Section 3 develops our model of the macroeconomy. Section 4 presents our main theoretical results. It also includes numerical simulations to illustrate how the model qualitatively matches several important empirical facts. Finally, Section 5 concludes.

2 Empirical Motivation

Our empirical motivation is divided into three sections. The first section documents and describes our data sources. The second illustrates a high level of reported competition between formal and informal firms and explores the degree to which this finding is robust to different measures, survey years, and regions. The final section investigates the relationship between competition, industry, and firm size (a proxy for productivity).

2.1 The Data

In order to motivate our theoretical model, we first highlight some salient empirical facts. We rely heavily on the survey data from the World Bank’s Enterprise Surveys.² These surveys are a stratified random sampling of manufacturing and retail firms with five or more employees in the formal sector. The surveys are conducted across a wide range of developing countries, and many countries have been surveyed multiple times in the data. We supplement the survey data with macroeconomic indicators from the World Bank and the United Nations Development Programme.

Our version of the Standardized Data contains countries surveyed from 2006 through mid-2016.³ In total we have data covering 140 different countries and over 124,000 firms. From the data we use four main survey questions to capture both competition and proxies for firm productivity. The first measure of competition is a binary indicator for whether a firm competes with informal firms. A second related measure asks firms the degree to which informal competition is an obstacle to their operations. We will refer to these measures as our binary and categorical measures

² This is one of the same data sources used by La Porta and Shleifer (2014).

³ Our data is the Standardized Data set released August 1st, 2016.

Table 1: Summary Statistics

Variable	Observations	Unweighted	Weighted
Binary Competition	104,560	50.9% (mean)	50.9% (mean)
Categorical Competition	118,677	1.0 (median)	
Firm Size (perm. full-time)	124,034	98.56 (mean)	65.59 (mean)
Firm Size (all full-time)	120,085	170.19 (mean)	123.73 (mean)

Observations exclude probable typos and non-responses. This standard is used throughout the paper unless otherwise noted . The “Weighted” column provides estimated means given the sampling weights and strata of the survey design. The sampling weights cannot be used to estimate a median, although in terms of proportions, roughly 50% of firms rank informal competition as a moderate obstacle or higher (a 2 or higher on a 0-4 scale). We drop several observations for temporary labor that are extreme outliers (10 standard deviations from the mean).

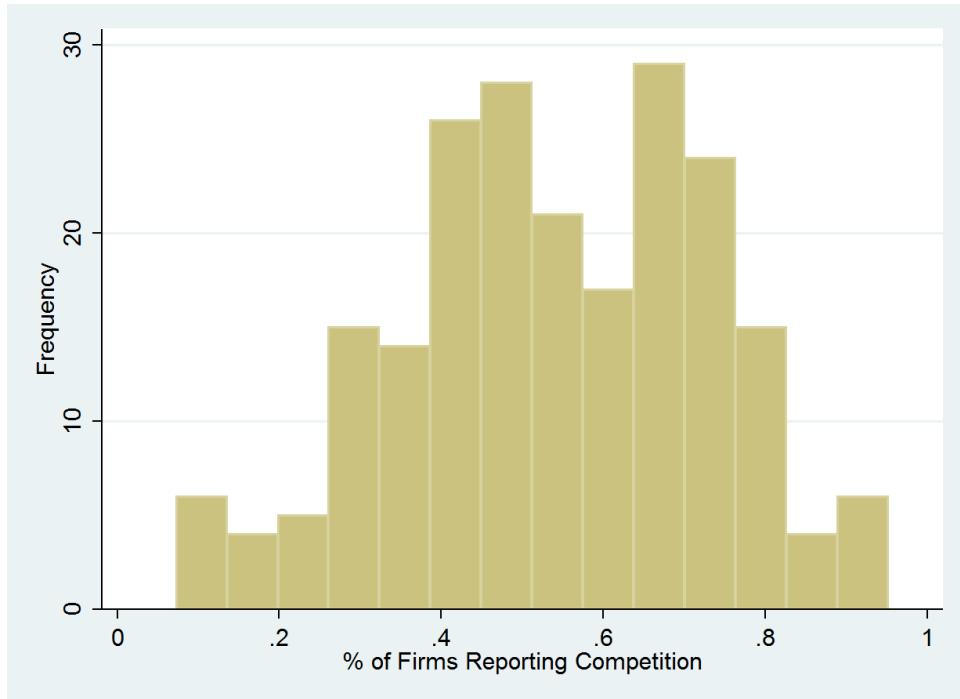
of competition, respectively. Finally, we use the reported number of permanent full-time employees and the number of full-time employees (permanent and temporary) to condition our findings on firm size and as a proxy for firm productivity. When appropriate, we utilize the Enterprise Survey’s probability weights so that individual firms can be properly weighted to reconstruct an approximation of the universe of manufacturing and retail firms in a given country.⁴ Table 1 provides pooled summary statistics for our variables of interest. The statistics for firm size illustrate the affect of weighting the data. Since small firms tend to be under-sampled, unconditional statistics with regard to the population will tend to be biased toward the characteristics of larger firms.

⁴ A more in-depth discussion of the appropriate use of these weights can be found in the implementation notes for each country-year survey. These notes can be found online at <http://www.enterprisesurveys.org>.

2.2 Competition

Table 1 indicates that 51% of formal sector firms reported that they faced such competition.⁵ Likewise, the median firm reported that informal firms were a minor obstacle, and roughly 25% of all firms surveyed (including non-applicable responses) reported informal firms were either a “major” or “very severe” obstacle to their operations. While the analysis that follows will further investigate the results presented in Table 1, the basic take-away remains clear: a significant percentage of formal sector firms face competition from informal firms.

Figure 1: Country-Level Means



The unit of observation is a country-year. All country means are weighted based on the probability weights provided by the World Bank Enterprise Survey. We drop all firms that did not answer either “yes” or “no” to the survey question.

⁵ Even if non-responses and typos are included as facing no competition from informal firms, about 43% of all firms say they compete.

Table 2: Competition: Cross-Country Regressions

Variable	(1)	(2)	(3)	(4)
Log GDP per capita	-0.051*** (0.011)	-0.054*** (0.011)	-0.043*** (0.013)	-0.041*** (0.013)
Time fixed effects	no	yes	no	yes
Region fixed effects	no	no	yes	yes
Constant	0.937*** (0.089)	1.039*** (0.090)	0.923*** (0.119)	0.831*** (0.133)
N	185	185	183	183

The dependent variable is the probability-weighted level of competition for each country surveyed. All regressions are estimated using OLS. Each country is put into one of five regions: Africa, Americas, Asia, Europe, or Oceania. Standard errors are provided in parenthesis. Asterisks denote significance at the 1% (***) , 5% (**) and 10% (*) levels for a standard two-tailed t-test.

Figure 1 shows the distribution of means across countries. While there is a broad range of levels of competition between the informal and formal sectors, all surveyed countries have some degree of competition between formal and informal firms. Table 2 presents a simple cross-country regression exercise that illustrates several key aspects of competition.⁶ First, competition is negatively related to income. While we are wary of making any causal inference, this correlations illustrates that our data is consistent with the dual view of informality. Second, a closer analysis of region and time effects shows that competition is remarkably stable across regions and time. See Appendix B for a complete regression table documenting that statistically insignificant role of time and most region dummies. On average, countries in the Americas tend to have slightly higher levels of competition, while countries in Asia tend to have slightly lower levels of competition.⁷

⁶ Note that Table 2 is abbreviated. The time and region fixed effects are broken out in Appendix B

⁷ The excluded case is countries in Oceania.

Finally, the degree of competition does not seem to be sensitive to the measure of competition. An analysis of the categorical measure of competition yields similar conclusions with regard to competition. Across countries and time, the average ranking is 1.33, meaning that the average formal sector firm surveyed sees informal competition as being a minor to moderate obstacle.⁸ This mean hides interesting heterogeneity and masks the severity of competition for many formal sector firms. Figure 2 shows the percentage of firms in each country that reported informal firms were a “very severe” (left panel) or “major” (right panel) obstacle to their firm.⁹

Between these two measures of competition, we prefer the binary measure. First, the categorical measure of competition is highly subjective. “Minor” may mean something very different for different firms. Second, the binary nature of the first measure allows for an easy interpretation of the country-level mean: the percentage of firms that compete with informal firms. This result is of importance because of the link it provides to productivity. Initially we had some concerns about the large number of missing observations (16.3% and 5.0% missing across the binary and categorical measures respectively). The percentage of observations missing is fairly constant across firm size.¹⁰ Further, since small firms are slightly more likely to be excluded and are more likely to report competition from informal firms (see next section), accounting for the missing values would likely increase the measured levels of completion. Subject to the necessary caveats with respect to survey data and measuring competition from only a formal perspective, we find the evidence quite compelling that formal and informal firms tend to compete in a meaningful way.¹¹

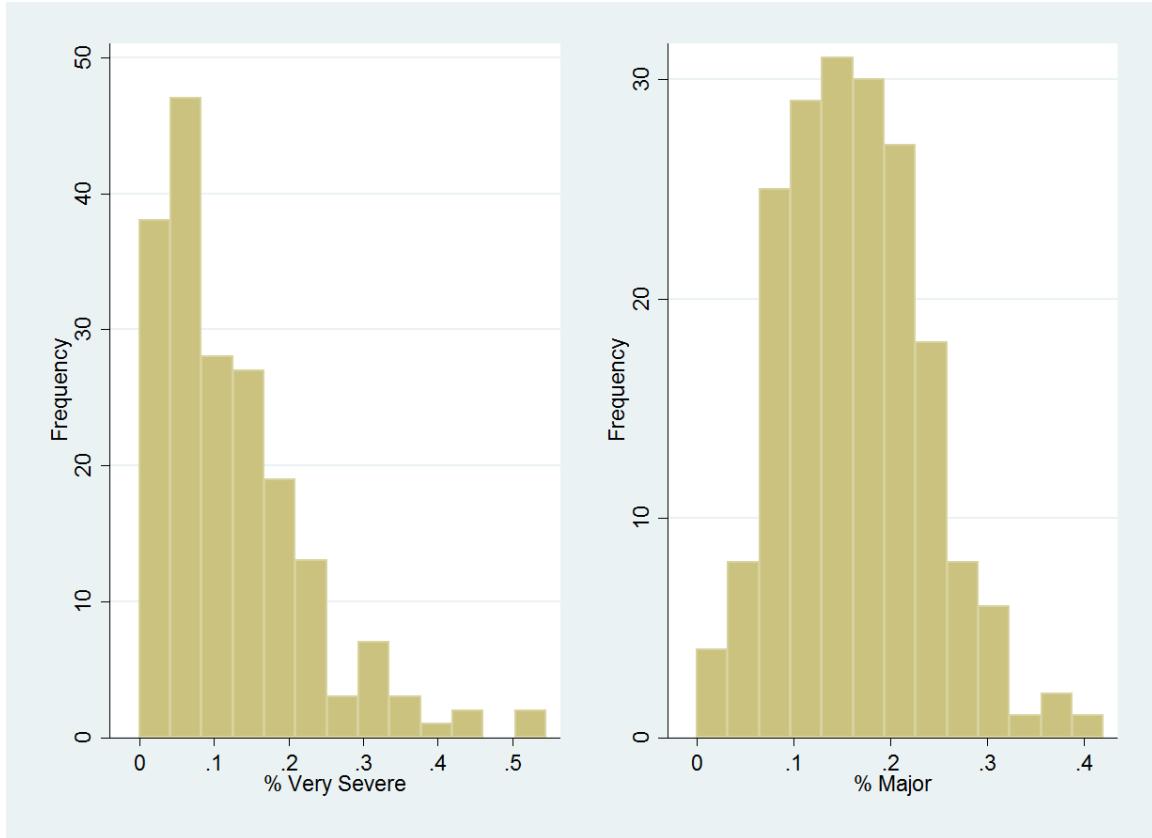
⁸ We recognize that interpreting the mean of such a subjective, categorical variable is problematic. Recognizing this, we tend to put more emphasis on the distribution of firm responses.

⁹ Note that “very severe” (4 out of 4) implies more problematic than “major.” (3 out of 4)

¹⁰ For instance, for our preferred measure of competition, 17.04% of small firms, 15.61% of medium firms, and 15.95% of large firms did not respond.

¹¹ Making inferences about informal firms based on survey data collected from formal sector firms

Figure 2: Degree of Competition across Countries



The unit of observation is a country-year. Individual firm surveys are weighted prior to aggregation. Note the vertical axis (frequency) is scaled differently in the left and right panels.

2.3 Firm Size and Competition

The Enterprise Survey data also offers evidence to suggest that smaller firms are more likely to compete with informal firms. Many theoretical papers assume a direct link between firm size and productivity. This is the case for instance in Melitz (2003). Empirical work as also tended to support this conclusion.¹² Throughout the remainder of the paper, we take the positive relationship between firm size and productivity as given and use them interchangeably. The negative relationship between firm size and competition is weaker than the previous conclusion that competition occurs. The strength of the relationship tends to hinge on the level of aggregation.

Pooling the data across countries and survey years and regressing our binary competition measure on firm size shows that firm size is negatively related to whether a firm competes with informal firms. Table 3 reports the weighted regression results across several different specifications. The linear probability model in column (3) suggest that increasing firm size by 100 employees would decrease the probability that a firm competes with informal firms by .41%. We prefer this specification due to its straightforward interpretation. The relatively small, although statistically strong, relationship should not be surprising given the wide range of idiosyncratic causes of firm competition for which we cannot control. The negative relationship is robust to other less involved specifications (see columns (1) and (2)) and different error term assumptions. Columns (4) and (5) estimate our preferred specification using a logit and probit regression respectively to account for the well known deficiencies of hypothesis testing with the linear probability model. Finally, these results are also robust to our more expansive definition of labor, permanent and temporary full-time workers. Those results are documented in Table 6 in Appendix C.

is not uncommon. See Dabla-Norris et al. (2008) for a typical example.

¹² For an example pertaining to the developing world see Söderbom and Teal (2004).

Table 3: Competition and Firm Size: Pooled Regressions

Variable	(1) OLS	(2) OLS	(3) OLS	(4) Logit	(5) Probit
Firm size	-0.0029*** (0.0010)	-0.0031** (0.0014)	-0.0042*** (0.0015)	-0.0242* (0.0144)	-0.0112*** (0.0041)
Log GDP/capita			-0.0422*** (0.0057)	-0.0520*** (0.0064)	-0.2186*** (0.0276)
Sector fixed effects	no	no	yes	yes	yes
Region fixed effects	no	no	yes	yes	yes
Time fixed effects	no	no	yes	yes	yes
Constant	0.5126*** (0.0070)	0.8805*** (0.0454)	0.9921*** (0.0811)	2.0684*** (0.3539)	1.2815*** (0.2183)
N	103,747	88,783	82,587	82,587	82,587

The dependent variable is the binary measure of whether a firm competes with informal firms. Firm size is measured as the number of permanent full-time employees. A one unit increase in firm size corresponds to an additional 100 workers as the variable has been scaled for aesthetic purposes. All regressions utilize probability weighting. Regional dummies are Africa, Americas, Asia, Europe, or Oceania. Sector fixed effects are based on two-digit ISIC codes. We drop industries that cannot be estimated with a logit/probit to maintain the same sample across specifications (3), (4), and (5). Standard errors are provided in parenthesis. Asterisks denote significance at the 1% (***)¹, 5% (**) and 10% (*) levels.

Table 4: Competition and Firm Size: Strata Means

Size Category	Mean
Small (<20)	55.6%
Medium (20-99)	49.2%
Large(100+)	42.4%
Total	50.9%

Means are unweighted as they are taken within the strata of the survey itself.

The patterns in Table 3 are more clearly visible when we examine means across firm size. To do so, we measure the mean value of our binary competition measure across the three strata for firm size: small, medium, and large. Table 4 reports the mean values across size categories. Clearly, as firm size increases, the average percentage of firms that report competing with informal firms falls.

Our review of the data leaves us with three motivating facts that must be explained or included as part of our theoretical model. First, the data is clear that at least some formal sector firms face competition from informal firms. In truth, a large percentage of formal sector firms likely face this type of competition. Second, the likelihood that a firm faces informal competition (or at least that is reports that it does) decreases with its size and productivity. Finally, any theory regarding formal and informal competition ought to roughly align with the dual view of informality, where informality decreases as countries develop.

3 Model

Our theoretical model builds on the framework developed by Melitz (2003), with two differences. First, we add a industry layer to the economy. This allows us to

examine how informal and formal competition vary across industry with different costs of entry. Second, we embed the choice for firms to be informal or formal into the model. We refer to this decision as firms' sectoral choice decision. These two deviations generalize the work of Melitz (2003) and build the necessary framework to examine whether formal and informal firms compete.

3.1 Model Set-Up

3.1.1 Households

Suppose there is a representative household that is endowed with income, Y . The household seeks to maximize its consumption, c , which is a Cobb-Douglas composite of goods produced in the \mathcal{I} industries of the economy. There is no subsistence level of consumption across industries. The household's problem can be written as

$$\begin{aligned} & \underset{C_i}{\text{maximize}} \quad c = \sum_{i=1}^{\mathcal{I}} \eta_i \log(C_i) \\ & \text{subject to} \quad \sum_{i=1}^{\mathcal{I}} P_i C_i = Y, \end{aligned}$$

where C_i is household consumption of goods from industry i , P_i is the price level in industry i and η_i captures the household's preferences over specific industries of the economy. We assume these preferences sum to one across all industries. The standard solution to the household's consumption problem across industries is

$$P_i C_i = \eta_i Y = R_i, \quad i \in \{1, \mathcal{I}\}, \tag{1}$$

where R_i is the total revenue of firms in industry i . Within each industry there is a mass of firms that produce distinct varieties, $\omega \in \Omega_i$. Industry production of the consumption good, C_i , and prices, P_i , are CES aggregates of the varieties produced in industry i :

$$C_i = \int_{\omega \in \Omega_i} (C_i(\omega)^\epsilon d\omega)^{\frac{1}{\epsilon}} \quad (2)$$

$$P_i = \int_{\omega \in \Omega_i} (p_i(\omega)^{1-\sigma} d\omega)^{\frac{1}{1-\sigma}}, \quad (3)$$

where $p_i(\omega)$ is the price of variety ω . Varieties are substitutes for each other such that $0 < \epsilon < 1$ and the elasticity of substitution between varieties is $\sigma = \frac{1}{1-\epsilon} > 1$.

Using equations (1) and (2) it can be shown that the demand for each variety is given by

$$C_i(\omega) = C_i \left(\frac{P_i}{p_i(\omega)} \right)^\sigma. \quad (4)$$

The household is indifferent between varieties that are produced by formal or informal firms.

3.1.2 Firms

Firms may decide to operate either formally or informally. Throughout the paper, firms operating informally are denoted with an I and firms operating formally with an F . Both types of firms *can choose* to operate in each industry i and enjoy market power by producing unique varieties, ω . All firms compete in monopolistically competitive markets.

It is worth emphasizing how our assumptions interface with the research question. Of central interest is how and whether formal and informal firms compete. Assuming that all firms are monopolistically competitive does not *ex ante* assume the answer to this question. In fact, our model is general enough to support a variety of views

of the informal economy. In particular, it could underscore the view that informal firms operate in their own economic space. In this way, informal firms would produce goods in the first n industries such that $i \in \{1, n\}$ and formal firms produce goods in the remaining industries $i \in \{n + 1, \mathcal{I}\}$ without loss of generality. In equilibrium both types of goods will exist, however, formal and informal firms would not directly compete within sectors.

3.1.3 Government

The government in the model plays two roles. First, it collects tax revenue from firms in the formal sector. Each formal sector firm is taxed at a rate τ of their profits. Second, the government locates and fines informal firms at a rate μ . Informal firms that are caught forfeit the entirety of their profits.¹³ The government's total revenue, T , is described by

$$T = \sum_{i=1}^{\mathcal{I}} \tau \int_{\omega \in F_i} \pi(\omega) d\omega + \mu \int_{\omega \in I_i} \pi(\omega) d\omega, \quad (5)$$

where $\pi()$ is the profitability of the firm producing good ω and F_i and I_i refer to set of firms operating in the formal and informal sectors in industry i , respectively.¹⁴ All revenue collected by the government is transferred back to the household as a lump-sum payment.

¹³ In an aggregate sense, μ could also be seen as capturing costs that are unique to the informal sector, for instance arbitrating disagreements without the help of formal legal institutions. In this sense, the total amount collected through enforcement (μ) would not go to the government, but be transfer directly to other households.

¹⁴ We assume that all firms are either entirely formal or entirely informal. Possible extensions to the model may allow formal firms to hide part of their operation or allow informal firms to avoid detection by paying bribes. While these aspects of informality are certainly of interest, they do no play a central role in determining whether formal and informal firms compete.

3.1.4 Aggregate Market Clearing

After accounting for taxes and fines, firms' net profits are transferred back to the household as dividend:

$$D = \sum_{i=1}^{\mathcal{I}} (1 - \tau) \int_{\omega \in F_i} \pi(\omega) d\omega + (1 - \mu) \int_{\omega \in I_i} \pi(\omega) d\omega. \quad (6)$$

Finally, market clearing implies that:

$$Y = wL + T + D = R. \quad (7)$$

Total wage labor, wL , is determined by a nominal wage rate w that is common to both formal and informal firms, and the labor supply L . Total income Y , in the economy is equivalent to total firm revenue. Note that Equations (1) and (7) imply the total revenue of each sector is fixed and is determined entirely by household income and preferences.

3.2 Firm Structure

3.2.1 Sectoral Entry

Each firm decides whether to enter industry i and produce its own variety ω . Entry into an industry requires the firm to pay an industry specific fixed cost f_i^e . For convenience we order industries such that

$$f_{i-1}^e < f_i^e. \quad (8)$$

After a firm enters an industry and pays the associated fixed cost, our model parallels Melitz (2003). All firms receive a productivity draw ϕ from a common distribution $g(\phi)$. This distribution is accompanied by the usual assumptions, mainly that $g(\phi)$ has support on the interval $(0, \infty)$, and we represent the continuous cumulative distribution for firm productivity as $G(\phi)$. Upon realizing its productivity, a firm may either decide to produce or immediately exit.

3.2.2 Production

Producing firms have access to an increasing returns production function that is the same across industries but differs based on whether a firm is formal or informal. As a result, the firm's labor demand function is:

$$l_i^s = f^s + \frac{C_i(\omega)}{\phi} \text{ for } s \in \{I, F\}, \quad (9)$$

where s denotes the firm's sectoral choice (informal or formal). It is assumed that $f^I < f^F$. This captures the idea that in addition to basic start-up costs faced by the firm, formal firms will also need to pay registration costs and fees in order to produce. Because all firms face the same residual demand curve, they choose a price equal to a constant mark-up over marginal cost:

$$p(\phi) = \frac{w}{\epsilon\phi}. \quad (10)$$

We proceed by normalizing the nominal wage rate such that $w = 1$.

Unlike Melitz (2003), the labor demand function in Equation (9) along with the pricing rule in Equation (10), imply that the firm's profits depend both upon their

productivity as well as their sector s . The expression for profits is given by

$$\pi_i^s(\phi) = \frac{R_i}{\sigma} (P_i \epsilon \phi)^{\sigma-1} - f^s \text{ for } s \in \{I, F\}. \quad (11)$$

However, it should be noted that the firm's revenue is independent of its formality decision:

$$r_i(\phi) = R_i (P_i \epsilon \phi)^{\sigma-1} \quad (12)$$

As in Melitz (2003), the ratios of two firms' outputs or revenues are equal to the ratios of their productivities:

$$\frac{C_i(\phi_1)}{C_i(\phi_2)} = \left(\frac{\phi_1}{\phi_2} \right)^\sigma, \quad \frac{r_i(\phi_1)}{r_i(\phi_2)} = \left(\frac{\phi_1}{\phi_2} \right)^{\sigma-1} \quad (13)$$

3.2.3 Cut-offs

Upon drawing a productivity, firms face two choices: whether to produce and in which sector to operate. Ultimately, the firms decisions will depend upon what productivity they draw. In order to streamline the exposition, this discussion will center around two lemmas that provide the main results, while their derivations can be found in the appendix. As in Melitz (2003) there is a large pool of possible entrants into each industry. As noted above, firms pay an industry specific fixed cost in order to receive a productivity draw from the cumulative distribution $G(\phi)$ (the distribution is the same across sectors). Once the firm receives their draw, they immediately decide whether or not to exit. Firms will exit if their profits are negative and because firm profits are increasing in the firm's productivity, there exists a productivity ϕ^* below which firms exit. This implies that the distribution of productivities among

operating firms is:

$$\gamma_i(\phi) = \begin{cases} \frac{g(\phi)}{1-G(\phi_i^*)} & \text{if } \phi \geq \phi_i^* \\ 0 & \text{otherwise} \end{cases}. \quad (14)$$

Therefore the average productivity of active firms is:

$$\tilde{\phi}(\phi_i^*) = \left[\frac{1}{1-G(\phi_i^*)} \int_{\phi_i^*}^{\infty} \phi^{\sigma-1} g(\phi) d\phi \right]^{\frac{1}{\sigma-1}}. \quad (15)$$

The firm's decision to enter and produce depend upon the profitability of production and the expected value of entry. As shown above a firm's profitability depends upon their productivity as well as their sector. This implies that the zero profit condition depends upon the structure of the industry.¹⁵ In contrast, because paying the fixed cost of entry takes place prior to the realization of productive and thus sectoral choice, there is a single free entry condition, regardless of the industry structure.

We differ from Melitz (2003) in how these conditions are calculated. Where Melitz (2003) relates average profit to the cut-off value for both the zero profit and free entry conditions, we are content to stay with the relationship between average revenue and the cut-off. The reason for this deviation is that in our model informality creates a non-convexity in the relationship between profits and firm productivity. In doing so, average profits depend on more than just the cut-off value; they also depend on the distribution of firm productivities. The choice to focus on average revenue means that we can remain distribution neutral. The follow lemma gives the expressions for the zero profit and free entry conditions.

Lemma 1. *The zero profit and free entry conditions can be written in terms of*

¹⁵In an industry with both formal and informal firms, firms do not require as high of revenues to be willing to produce as they have the option of paying the lower fixed costs associated with informality.

average revenue as a function of the minimum productivity level, ϕ^* .

$$\bar{r}_i = \left(\frac{\tilde{\phi}(\phi_i^*)}{\phi_i^*} \right)^{\sigma-1} \sigma f^s \quad (ZPC)^{16} \quad (16)$$

$$\bar{r} = \frac{\sigma(f^e + (G(\bar{\phi}) - G(\phi^*))(1 - \mu)f^I + (1 - G(\bar{\phi}))(1 - \tau)f^F}{(G(\bar{\phi}) - G(\phi^*))(1 - \mu) \left[\frac{\tilde{\phi}^I}{\bar{\phi}} \right]^{\sigma-1} + (1 - G(\bar{\phi}))(1 - \tau) \left[\frac{\tilde{\phi}^F}{\bar{\phi}} \right]^{\sigma-1}} \quad (FE) \quad (17)$$

Note that the (ZPC) found in equation (16) is dependent upon the structure of industry i . In a mixed industry (one with both informal and formal firms), $s = I$, while in an industry with only formal firms, $s = F$. The minimum productivity cut-off, ϕ_i^* , is determined by the intersection of the free entry and zero profit conditions. However, without additional assumptions regarding the distribution of productivities it is not possible to determine where the intersection occurs, if it occurs at all. The next section will place additional structure on $G(\cdot)$ and derive the main results of the paper.

The second cut-off regarding formality is determined by looking at the expected profit from operating in each sector. In particular, the formality cut-off occurs when $E\pi_i^I(\bar{\phi}) = \pi_i^F(\bar{\phi})$. Lemma 2 gives the result below.

Lemma 2. *Suppose that within industry i there are both informal and formal firms. There exists a productivity level, $\bar{\phi}_i$, such that firms who draw a productivity level ϕ greater than $\bar{\phi}_i$ enter the formal sector. Moreover, this formality cut-off, $\bar{\phi}_i$, can be explicitly written as:*

$$\bar{\phi}_i = \left[\frac{\sigma((1 - \tau)f^F - (1 - \mu)f^I)}{(\mu - \tau)R_i} \right]^{\frac{1}{\sigma-1}} (P_i \epsilon)^{-1} \quad (18)$$

Clearly, we require $\tau < \mu \leq 1$, otherwise all firms would become informal. Also

it is straightforward to show that if $\mu = 1$, then the formality cut-off is the same as the zero profit condition, which means that all firms within an industry would be formal.

3.2.4 Aggregation

Firm level prices and productivities can be aggregated in the same manner as Melitz (2003). The industry price given in Equation (3) can be written as:

$$P_i = \left[\int_0^\infty p(\phi)^{1-\sigma} M_i \gamma_i(\phi) d\phi \right]^{\frac{1}{1-\sigma}}, \quad (19)$$

where M_i is the mass of firms operating in industry i . Using Equations (19) and (15) the sector specific aggregate price level, quantity produced, revenue, and profits can be written as:

$$\begin{aligned} P_i &= M_i^{\frac{1}{1-\sigma}} p(\tilde{\phi}_i), & R_i &= P_i C_i = M_i r_i(\tilde{\phi}_i) \\ C_i &= M_i^{\frac{1}{\epsilon}} C_i(\tilde{\phi}_i) \end{aligned}$$

In an effort to conserve notation, the industry indicator i will be dropped for the following discussion.

4 Results

4.1 Unique Solutions

Before looking at the equilibrium of the model, we want to discuss a couple of results that are unique for a given ϕ^* . First, note that the average productivity given in Equation (15) is unique for a given ϕ^* . This implies that average revenue $r(\tilde{\phi}(\phi^*)) = \bar{r}$

is unique in the cut-off value as well. Using \bar{r} , the mass of firms can pinned down: $M = R/\bar{r}$. The price level can be determined using average revenue. A firm with the average productivity in the sector has revenue given by: $r(\tilde{\phi}(\phi^*))$, which must be equal to \bar{r} , therefore:

$$R(P\epsilon\tilde{\phi}(\phi^*))^{\sigma-1} = \left[\frac{\tilde{\phi}(\phi^*)}{\phi^*} \right]^{\sigma-1} \sigma f^s.$$

This expression can be solved for the industry price level P :

$$P = \frac{1}{\epsilon\phi^*} \left(\frac{\sigma f^s}{R} \right)^{\frac{1}{\sigma-1}} \quad (20)$$

which is clearly unique for a given ϕ^* . Finally, plugging Equation (20) into (18) yields:

$$\bar{\phi} = \hat{F}_s \phi^* \quad (21)$$

where

$$\hat{F}_s = \left[\frac{(1-\tau)f^F - (1-\mu)f^I}{(\mu-\tau)f^s} \right]^{\frac{1}{\sigma-1}} \geq 1.$$

This shows that not only is the formal cut-off unique for a given ϕ^* but it also directly proportional to it. It should be noted that where the superscript s indicates whether a firm is formal or informal, the subscript s indicates the distribution of firm type within an industry. In the case of \hat{F} , when $s = I$ this means that the industry includes both informal and formal firms (referred to throughout the text as a mixed industry), whereas, when $s = F$ the industry only includes formal firms. It should be noted that $\hat{F}_s = 1$ and it only occurs when the probability of being caught when informal is $\mu = 1$ or when the fixed costs are the same between sectors ($f^F = f^I$).

4.2 Equilibrium

4.2.1 Special Case: Pareto Distribution

This section will analytically derive the main results of the model using the Pareto distribution, while the following section will numerically show that these results hold for other well behaved distributions.

We assume the distribution of productivity draws is Pareto with minimum value k and shape parameter α . The first step in solving the model is to solve for the average productivities for the entire industry as well as for informal and formal firms, which leads to the following lemma:

Lemma 3. *The average productivities for an entire industry and for formal and informal firms within an industry are directly proportional to the cut-off value, ϕ^* , and are given by*

- $\tilde{\phi} = \left(\frac{\alpha}{1+\alpha-\sigma}\right)^{\frac{1}{\sigma-1}} \phi^*$,
- $\tilde{\phi}^I = \left[\left(1 - \hat{F}_j^\alpha\right)^{-1} \left(1 - \hat{F}_j^{-1-\alpha+\sigma}\right) \left(\frac{\alpha}{1+\alpha-\sigma}\right)\right]^{\frac{1}{\sigma-1}} \phi^*$,
- $\tilde{\phi}^F = \left(\frac{\alpha}{1+\alpha-\sigma}\right)^{\frac{1}{\sigma-1}} \hat{F}_j \phi^*$.

All proofs can be found in the appendix.¹⁷ From Lemma 3 we can determine how the zero profit condition and the free entry condition depend on the cut-off value ϕ^* . The result is given in Proposition 4.

Proposition 4. *If the underlying distribution of firm productivities is Pareto with minimum value k and shape parameter α , then the zero profit condition is horizontal, and it is given by:*

$$\bar{r}_s = \left(\frac{\alpha}{1+\alpha-\sigma}\right) \sigma f^s \quad (ZPC). \quad (22)$$

¹⁷ In order to ensure that the cut-offs are positive we assume that $\sigma < 1 + \alpha$

In contrast, the free entry condition is upward sloping and is given by:

$$\bar{r}_s = \sigma \left(\frac{\phi^*}{k} \right)^\alpha \left(\frac{f^e}{\bar{F}_s} \right) + \sigma f^s \quad (FE) \quad (23)$$

where

$$\bar{F}_s = \left(1 - \hat{F}_s^{-1-\alpha+\sigma} \right) (1 - \mu) + \hat{F}_s^{-1-\alpha+\sigma} (1 - \tau).$$

Proof. The proof is simple an application of Lemma 3. \square

Using Proposition 4 it is possible to calculate the cut-off value ϕ^* :

$$\phi_s^* = k \left(\frac{f^s \bar{F}_s (\sigma - 1)}{f^e (1 + \alpha - \sigma)} \right)^{\frac{1}{\alpha}} \quad (24)$$

Note that in order to ensure that $\phi_s^* > k$, we assume that the fixed cost of entry is sufficiently low.¹⁸ Not surprisingly, the cut-off value is increasing in the fixed cost of production, as greater fixed costs require more productive firms to cover them. Additionally, the higher cost of entry f^e results in a lower cut-off value. This is because the higher costs lower the number of entrants, resulting in less competition and higher prices. This allows lower productivity firms to enter and produce with positive profits.

Now that we have solved for the equilibrium cut-off value, it is possible to answer two key questions. First, how does the equilibrium with informal and formal firms differ from an equilibrium with only formal firms? This will allow us to understand the impact that informal firms have on the economy as a whole. The second question is in regards to the empirical relationship between formal and informal firms. In

¹⁸ We require that

$$f^e < \frac{f^s \bar{F}_s (\sigma - 1)}{1 + \alpha - \sigma}.$$

particular, do we see an overlap between formal and informal firms as in the data? Starting with the first question, we proceed with Proposition 5.

Proposition 5. *Suppose that we have two economies that share the same fixed costs, revenues, and tax rates, but they differ in their probability of closing and fining informal firms. In the first economy (economy \mathcal{F}), $\mu = 1$ such that there are no informal firms. In the second, economy (economy \mathcal{I}), $\mu < 1$ such that there are both formal and informal firms within an industry.*

1. $\phi_{\mathcal{I}}^* < \phi_{\mathcal{F}}^* < \bar{\phi}_{\mathcal{I}}$
2. $\bar{r}_{\mathcal{I}} < \bar{r}_{\mathcal{F}}$
3. $M_{\mathcal{I}} > M_{\mathcal{F}}$
4. $M_{\mathcal{I}}^F < M_{\mathcal{F}}^F$

Proof. See Appendix A. □

These results describe the main effects of informality. First, Proposition 5.1 shows that informality allows lower productivity firms to enter into the market. This is a result of the lower fixed costs associated with being informal, those productivity draws that previously could not afford to enter the market, now can. However, the effect of having lower productivity firms entering the market is that the cut-off to become formal increases. Firms need a higher productivity draw to enter into the formal sector. The reason for this comes from Proposition 5.2, which shows that informal firms cause average revenue to fall. The fall in revenue due to increased competition (Proposition 5.3) makes it more difficult to cover the fixed costs of the formal sector, which in turn shrinks the formal sector relative to the economy with no informal competition (Proposition 5.4).

One the empirical results from the survey data was that smaller formal firms indicated that they faced more competition from informal firms. In order to see how

well the model addresses this observation, we will look at the percentage of profit lost as a result of informal competition. Explicitly, for a given productivity ϕ , the percentage of profit loss due to informal competition can be calculated as:

$$PPL(\phi) = \frac{\pi^F(\phi) - \pi^I(\phi)}{\pi^F(\phi)} \quad (25)$$

which brings us to our next proposition.

Proposition 6. *The percentage of profit lost due to informality is declining in firm productivity.*

Proof. See Appendix A. □

Proposition 6 states that more productive (larger) firms lose a smaller share of their profits as a result of informal competition. This is consistent with the data as these firms are also less likely to indicate that they compete against informal firms. The reason for this result, is that larger more productive firms charge lower prices than their competition. When smaller, less productive informal firms enter the market they charge higher prices posing little competitive threat to larger more productive firms. However, those firms on the margin between formal and informal face strong competition because their prices will be very similar.

The final question we want to answer is whether there are overlapping productivity distributions. Empirically, there appears to be an overlap between informal and formal firms when we look at measures of total factor productivity (TFP). Based on the model set-up it is clear that within an industry, the distinct cut-offs mean that there would be no overlap similar to the theoretical results of Rauch (1991). However, it is possible for there to be overlap across industries, which brings us to the following proposition.

Proposition 7. Suppose that there are two industries denoted 1 and 2 and they have different costs of entry. Without loss of generality, assume that: $f_1^e < f_2^e$. Under this assumption, the ratio of cut-off productivities is:

$$\frac{\phi_1^*}{\phi_2^*} = \frac{f_2^e}{f_1^e}$$

If the ratio of entry costs is less than \hat{F} , then there is overlap of formal and informal TFP across industries.

Proof. See Appendix A. □

Proposition 7 effectively says that if the cut-off values are not significantly different, then there exists a range of productivities that would cause a firm to be formal in industry 2 but informal in industry 1.

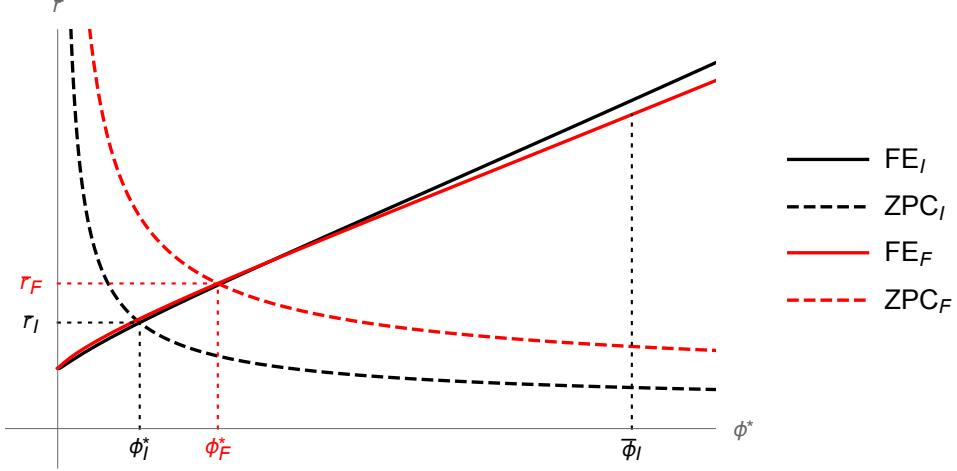
4.2.2 General

Now that the results of the model have been fully characterized analytically using the Pareto distribution it is time to turn to generalizing the results to other distributions. The figures below are created using a log-normal distribution but the results are robust to a wide-range of “well-behaved” distributions (exponential, gamma, Weibull) and parameter choices.¹⁹ The aim of this section of the paper is to show that the results that were analytically derived under the Pareto distribution are robust to other distributions as well.

Starting with Proposition 4, Figure 4 below graphs the free entry and zero profit cut-off lines for two economies: one with both informal and formal (mixed economy) firms and the other with only formal firms.

¹⁹ We did not come across a set of parameters that qualitatively changed the results in presented in this section, despite our honest attempts to do so.

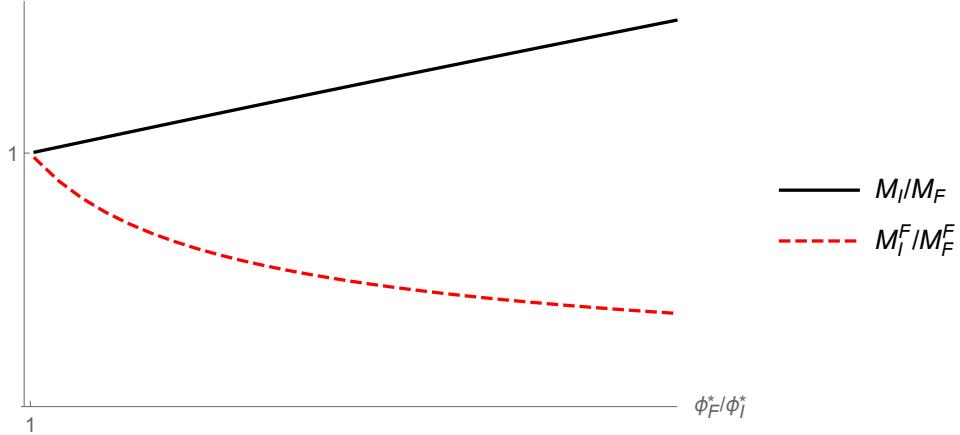
Figure 3: Equilibrium cut-off (ϕ^*) and average revenue (\bar{r})



The first two results of proposition 5 are readily apparent. The mixed economy has a lower cut-off and average revenue than the economy without informal firms. In addition, based on average revenue it is clear that the mass of firms is smaller in the economy with only formal firms. These results together confirm the underlying intuition of the model, informality allows those firms who otherwise be priced on the market to enter. The entry of additional firms creates increased competition, resulting in lower revenues on average. The one result that this figure cannot speak to however, is the relative size of the formal sector in each economy. This result is given in the figure below. It should be noted that the figure below is generated by varying the fixed cost associated with being formal within the range $\{f^I, 1.5 * f^I\}$. The ratios of the mass of firms (M_I/M_F) and the mass of formal firms (M_I^F/M_F^F) are plotted against the ratio of the cut-off values (ϕ_F^*/ϕ_I^*). It is expected that the ratio of the cut-offs would rise as the fixed cost of formality rises relative to the fixed cost of being informal.

Not surprisingly, the mass of firms in the mixed economy rises relative to the

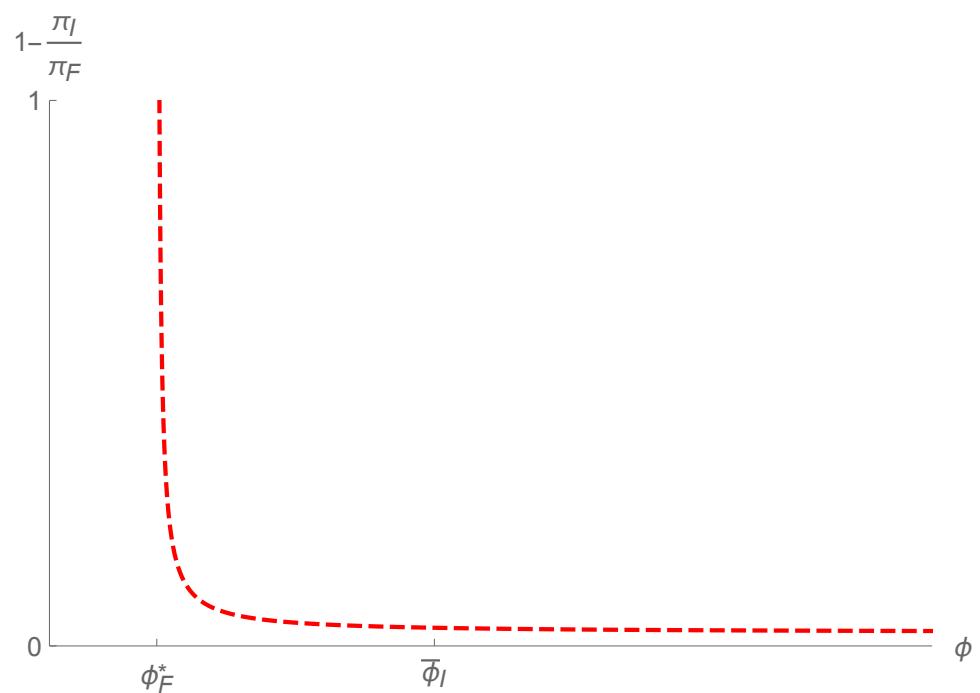
Figure 4: The ratios of the mass of firms (M_I/M_F) and the mass of formal firms (M_I^F/M_F^F) is plotted against the ratio of the cut-off values (ϕ_F^*/ϕ_I^*) for an economy with both formal and informal firms (I) and with only formal firms (F).



economy with only formal firms as f^F increases. This is because as the fixed cost of formality increases it lowers the number of formal firms. In the formal only economy, firms that do not become formal exit (lowering the over all number of firms), while in the mixed economy these firms simply become informal. The numerical results confirm that the mixed economy will see very little change in the mass of firms while the formal only economy will see a significant decline. What is particularly interesting in this figure is that despite the fact that the mass of firms in the formal only economy is falling relative to the mixed economy, the mass of formal firms is relatively increasing. This means that as the fixed costs of formality increase it causes more firms to abandoned formality in the mixed economy than those who exit in the formal only economy.

Proposition 6 showed that the firms that were most affected by informal competition were the ones with relatively low productivity. In Figure 5 below, we consider this result more generally. The metric of increased competition continues to be the percentage of profit lost due to informal firms. Consistent with what was shown

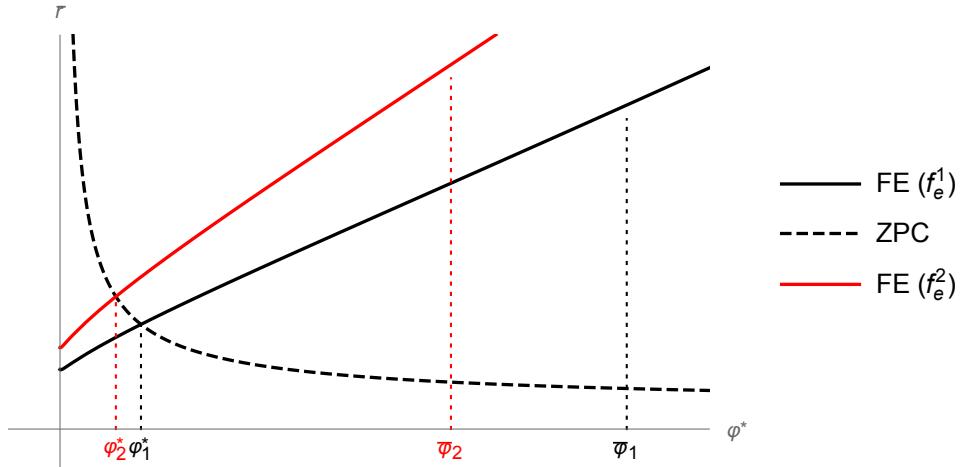
Figure 5: Percentage of profit loss due to informal competition



under the Pareto distribution, the percentage of profit loss is falling in firm productivity. The most significant losses occur for those firms who would be near the cut-off in a formal only economy, however it should be noted that these firms would no longer be formal in a mixed economy. It is still clear that those firms would be formal regardless of the structure of the economy (those with $\phi > \bar{\phi}_I$), the least productive ones are the most affected by informal competition.

The final proposition in the previous section showed that between industries there is a TFP overlap between formal and informal firms. Figure 6 generalizes this result for other distributions. To see this suppose there are two industries with different costs of entry (WLOG assume that $f_e^1 < f_e^2$). In Figure 6, the productivity range

Figure 6: TFP overlap across industries



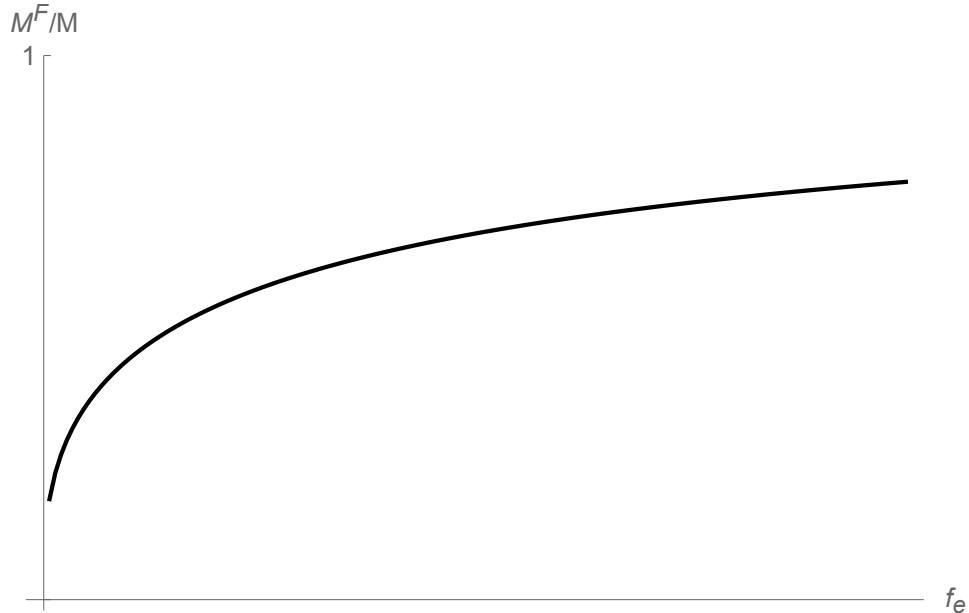
$\{\bar{\phi}_2, \bar{\phi}_1\}$ is comprised of formal firms in industry 2 and informal firms in industry 1. The reason for this overlap is that the higher cost of entry, shrinks the mass of firms, thus lowering competition and raising the overall price level. The higher price level means that less productive firms can cover the costs of formality. One of the questions that arises from Figure 6 is how does the percentage of firms that are

formal change with the fixed cost of entry. Figure 6 shows that increasing the cost of entry results in a lower cut-off, but it is unclear how this changes the distribution of formal and informal firms. In order to see how the distribution changes, we solve for the percentage of firms that are formal:

$$\frac{M^F}{M} = \frac{\tilde{\phi}^{\sigma-1} - (\tilde{\phi}^I)^{\sigma-1}}{(\tilde{\phi}^F)^{\sigma-1} - (\tilde{\phi}^I)^{\sigma-1}}.$$

Because $\tilde{\phi}^F > \tilde{\phi}$ the expression above is clearly between zero and one. Figure 7 below shows how the percentage of formal firms changes as the cost of entry increases.

Figure 7: Percent of firms that are formal plotted against the cost of entry.



Clearly, the percentage of firms that are formal is increasing with the fixed cost. This result follows the same intuition for why the cut-off value falls under higher fixed costs. Ultimately, higher fixed costs lower the amount of competition, resulting in higher prices. In turn, higher prices allow more firms to become formal.

5 Conclusion

The principle purpose of this paper is to determine the nature of competition between formal and informal firms in developing countries. Despite the central importance of this question, the literature has yet to reach a consensus about whether formal and informal firms operate in different markets. Our theoretical model shows that under very general conditions, there will be industries where both informal and formal sector firms compete. This attribute of the model is corroborated by strong survey evidence that formal firms face competition from informal firms. We also show that across sectors, there will be an overlap in the productivity distributions of formal and informal sector firms. These two principle findings reconcile the conflicting results of Rauch (1991) and the empirical evidence cited in Nataraj (2011) among others.

The model also matches the data in that it predicts that higher productivity firms suffer less loss in profits from informal competition. Put more succinctly, they face less competition from informal firms. This result builds into the dual view of informality, a view that we believe is supported by our findings. As countries develop among a variety of margins, informality decreases. The data that we use to motive the model is consistent with previous research by La Porta and Shleifer (2014) that showed informality decreases with income. The model also makes connections to important work that emphasizes the quality of institutions, such as Dabla-Norris et al. (2008), by showing that informality decreases as the ability of the government to enforce formality increases.

The development of this model opens a pathway for future research on informality. In particular, by better understanding how firms compete, we can begin to investigate the role of policy on existing firms and the decision of new entrepreneurs to enter the formal or informal sectors. The dual view of informality continues to be the

best description of the role that informal firms play in the process of development. Our research is most important in that it clarifies how those informal firms interact with the formal sector, and it provides a mechanism to examine the role of policy in speeding the process of development.

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Appendix A Proofs

A.1 Proof of Lemma 3

Proof. First note that:

$$1 - G(\phi) = \left(\frac{k}{\phi} \right)^\alpha$$

$$G(\bar{\phi}) - G(\phi^*) = \left(\frac{k}{\phi^*} \right)^\alpha - \left(\frac{k}{\bar{\phi}} \right)^\alpha = \left(\frac{k}{\phi^*} \right)^\alpha - \left(\frac{k}{\hat{F}_j \phi^*} \right)^\alpha = \left(\frac{k}{\phi^*} \right)^\alpha \left(1 - \hat{F}_j^{-\alpha} \right)$$

This implies that:

$$\begin{aligned} \tilde{\phi} &= \left[\left(\frac{\phi^*}{k} \right)^{-\alpha} \int_{\phi^*}^{\infty} \phi^{\sigma-1} g(\phi) d\phi \right]^{\frac{1}{\sigma-1}} = \left[\left(\frac{\phi^*}{k} \right)^{-\alpha} \left(\frac{\alpha k^\alpha \phi^{-1-\alpha+\sigma}}{1+\alpha-\sigma} \right) \right]^{\frac{1}{\sigma-1}} = \left(\frac{\alpha}{1+\alpha-\sigma} \right)^{\frac{1}{\sigma-1}} \phi^* \\ \tilde{\phi}^I &= \left[\left(\frac{k}{\phi^*} \right)^{-\alpha} \left(1 - \hat{F}_j^{-\alpha} \right)^{-1} \int_{\phi^*}^{\bar{\phi}} \phi^{\sigma-1} g(\phi) d\phi \right]^{\frac{1}{\sigma-1}} \\ &= \left[\left(\frac{k}{\phi^*} \right)^{-\alpha} \left(1 - \hat{F}_j^{-\alpha} \right)^{-1} \left(\frac{\alpha k^\alpha (\phi^*)^{-1-\alpha+\sigma}}{1+\alpha-\phi} \right) \left(1 - \hat{F}_j^{-1-\alpha+\sigma} \right) \right]^{\frac{1}{\sigma-1}} \\ &= \left[\left(1 - \hat{F}_j^{-\alpha} \right) \left(1 - \hat{F}_j^{-1-\alpha+\sigma} \right) \left(\frac{\alpha}{1+\alpha-\sigma} \right) \right]^{\frac{1}{\sigma-1}} \phi^* \\ \tilde{\phi}^F &= \left[\left(\frac{k}{\bar{\phi}} \right)^{-\alpha} \int_{\bar{\phi}}^{\infty} \phi^{\sigma-1} g(\phi) d\phi \right]^{\frac{1}{\sigma-1}} = \left[\left(\frac{k}{\bar{\phi}} \right)^{-\alpha} \left(\frac{\alpha k^\alpha \bar{\phi}^{-1-\alpha+\sigma}}{1+\alpha-\sigma} \right) \right]^{\frac{1}{\sigma-1}} = \left(\frac{\alpha}{1+\alpha-\sigma} \right)^{\frac{1}{\sigma-1}} \hat{F}_j \phi^* \end{aligned}$$

□

A.2 Proof of Proposition 5

Proof. 1. The proof for the first inequality follows from the ratio of ϕ_F^*/ϕ_I^* :

$$\frac{\phi_F^*}{\phi_I^*} = \left(\left(\frac{f^I(1-\mu)}{f^F(1-\tau)} \right) + \hat{F}_I^{-\alpha} \left(1 - \frac{f^I(1-\mu)}{f^F(1-\tau)} \right) \right)^{-\frac{1}{\alpha}} > 1$$

where the inequality follows from the fact that $\left(\frac{f^I(1-\mu)}{f^F(1-\tau)} \right) < 1$ and $\hat{F}_I^{-\alpha} < 1$.

The second inequality comes from the ratio of $\bar{\phi}_I/\phi_F^*$:

$$\frac{\bar{\phi}_I}{\phi_F^*} = \hat{F}_I \left(\left(\frac{f^I(1-\mu)}{f^F(1-\tau)} \right) + \hat{F}_I^{-\alpha} \left(1 - \frac{f^I(1-\mu)}{f^F(1-\tau)} \right) \right)^{\frac{1}{\alpha}} > 1$$

To see where the inequality comes from, let $x = \left(\frac{f^I(1-\mu)}{f^F(1-\tau)} \right)$, then we need:

$$\hat{F}_I((x + \hat{F}_I^{-\alpha}(1-x))^{1/\alpha} > 1$$

which can be rearranged to be:

$$x + \hat{F}_I^{-\alpha}(1-x) > \hat{F}_I^{-\alpha}$$

rearranging one more time, yields:

$$\hat{F}_I^{-\alpha} < 1$$

which is unambiguously true.

2. This result comes about by taking the ratio of the revenues in each economy:

$$\frac{\bar{r}_F}{\bar{r}_I} = \frac{f^F}{f^I} > 1$$

3. We know that the mass of firms equals $M = R/\bar{r}$, then this result is simply an application of the second result.
4. First, we know that total revenue is equal to average revenue times the mass of firms:

$$M\bar{r} = R$$

This can also be written as the sum of the total revenue in each sector:

$$M^I\bar{r}^I + M^F\bar{r}^F = R$$

Note that $M = M^I + M^F$, which means the above equation can be written as:

$$(M - M^F)\bar{r}^I + M^F\bar{r}^F = R$$

Solving for M^F yields:

$$M^F = \frac{R \left(1 - \frac{\bar{r}^I}{\bar{r}} \right)}{\bar{r}^F - \bar{r}^I}$$

where the substitution $M = R/\bar{r}$ is made. Using the definitions of \bar{r}, \bar{r}^I , and \bar{r}^F above, we can write the mass of firms in the formal sector as:

$$M^F = \hat{F}^{-\alpha} \frac{R}{\bar{r}}$$

To get the result above, take the ratio of M^F in each economy:

$$\frac{M_I^F}{M_F^F} = \frac{\frac{R}{\bar{r}_I} \hat{F}_I^{-\alpha}}{\frac{R}{\bar{r}_F}} = \frac{f^F}{f^I} \hat{F}_I^{-\alpha} < 1$$

where the inequality can be easily shown using the definition of \hat{F}_I . □

A.3 Proof of Proposition 6

Proof. First note that a firm's profit can be written as:

$$\pi_s^j(\phi) = \frac{r_s^j(\phi)}{\sigma} - f^j$$

We know that a firm's revenue can be expressed as a function of the cut-off value ϕ_s^* :

$$r_s(\phi) = \left(\frac{\phi}{\phi_s^*} \right)^{\sigma-1} r(\phi_s^*) = \left(\frac{\phi}{\phi_s^*} \right)^{\sigma-1} \sigma f^s$$

Therefore the ratio of revenue for a given ϕ across economies is:

$$\frac{r_F(\phi)}{r_I(\phi)} = \frac{\left(\frac{\phi}{\phi_F^*} \right)^{\sigma-1} \sigma f^F}{\left(\frac{\phi}{\phi_I^*} \right)^{\sigma-1} \sigma f^I} = \left(\frac{\phi_F^*}{\phi_I^*} \right)^{\sigma-1} \frac{f^F}{f^I} \equiv \chi > 1$$

Note that the revenue in an economy with only formal firms is proportional to the revenue in an economy with informal and formal firms. To see how we get the inequality, substitute in for ϕ_s^* and rearrange to get:

$$\left(\frac{1-\tau}{\mu-\tau} - \frac{f^I(1-\mu)}{f^F(\mu-\tau)} \right)^{-\frac{\alpha}{\sigma-1}} \left(1 - \frac{f^I(1-\mu)}{f^F(1-\tau)} \right) > 1 - \left(\frac{f^F}{f^I} \right)^{\frac{1+\alpha-\sigma}{\sigma-1}} \left(\frac{1-\mu}{1-\tau} \right)$$

Define the RHS and LHS as:

$$LHS = \left(\frac{1-\tau}{\mu-\tau} - \frac{f^I(1-\mu)}{f^F(\mu-\tau)} \right)^{-\frac{\alpha}{\sigma-1}} \left(1 - \frac{f^I(1-\mu)}{f^F(1-\tau)} \right)$$

$$RHS = 1 - \left(\frac{f^F}{f^I} \right)^{\frac{1+\alpha-\sigma}{\sigma-1}} \left(\frac{1-\mu}{1-\tau} \right)$$

Note that when $f^F = f^I$, $LHS = RHS = \frac{\mu-\tau}{1-\tau}$. Taking the derivative of each side with respect to f^F yields:

$$\frac{\partial LHS}{\partial f^F} < 0, \quad \frac{\partial RHS}{\partial f^F} < 0$$

However:

$$\frac{\partial RHS/\partial f^F}{\partial LHS/\partial f^F} > 1$$

Therefore the RHS is falling faster than the LHS, therefore

$$LHS > RHS \quad \forall f^F > f^I$$

Going back to the ratio of the two revenues, it implies that the percentage of profit loss is equal to:

$$PPL^j(\phi) = \frac{r_I(\phi)(\chi-1)}{\chi r_I(\phi) - f^j}$$

Taking the derivative w.r.t. ϕ yields:

$$\frac{\partial PPL(\phi)}{\partial \phi} = -\frac{r'_I(\phi)(\chi-1)f^j}{(\chi r_I(\phi) - f^j)^2} < 0$$

where $r'_I(\cdot) < 0$.

□

A.4 Proof of Proposition 7

Proof. If $f_e^2/f_e^1 < \hat{F}$, then it is clear that:

$$\phi_2^* < \phi_1^* = \frac{f_e^2}{f_e^1} \phi_2^* < \hat{F} \phi_2^* < \hat{F} \phi_1^*$$

□

Appendix B Competition: Cross-Country Regressions

Table 5: Competition: Cross-Country Regressions

Variable	(1)	(2)	(3)	(4)
Log GDP per capita	-0.051*** (0.011)	-0.054*** (0.011)	-0.043*** (0.013)	-0.041*** (0.013)
Time fixed effects				
2007	-0.078			0.063
2008	-0.161**			0.097
2009	-0.082*			0.086*
2010	-0.066			0.016
2011	-0.113			0.034
2012	-0.106			0.168
2013	-0.149***			0.042
2014	-0.083			0.058
2015	-0.206***			0.024
Region fixed effects				
Africa		0.012		0.040
Americas		0.099		0.159*
Asia		-0.182***		-0.171**
Europe		-0.106		-0.098
Constant	0.937*** (0.089)	1.039*** (0.090)	0.923*** (0.119)	0.831*** (0.133)
N	185	185	183	183

The dependent variable is the probability-weighted level of competition for each country surveyed. All regressions are estimated using OLS. Each country is put into one of five regions: Africa, Americas, Asia, Europe, or Oceania. Standard errors are provided in parenthesis. All coefficients are relative to a country from Oceania in 2006. Asterisks denote significance at the 1% (***)¹, 5% (**)² and 10% (*) levels for a standard two-tailed t-test.

Appendix C Additional Firm Size and Competition Results

Table 6: Competition and Firm Size: Pooled Regressions

Variable	(1)	(2)	(3)	(4)	(5)
	OLS	OLS	OLS	Logit	Probit
Firm size	-0.0034** (0.0014)	-0.0030** (0.0013)	-0.0042*** (0.0014)	-0.0243* (0.0110)	-0.0113*** (0.0041)
Log GDP/capita			-0.0403*** (0.0058)	-0.0432*** (0.0061)	-0.1823*** (0.0264)
Sector fixed effects	no	no	yes	yes	yes
Region fixed effects	no	no	yes	yes	yes
Time fixed effects	no	no	yes	yes	yes
Constant	0.5178*** (0.0072)	0.8659*** (0.0461)	0.9423*** (0.0805)	1.9714*** (0.3508)	1.1546*** (0.2164)
N	99,349	88,482	78,621	78,621	78,621

The dependent variable is the binary measure of whether a firm competes with informal firms. Firm size is measured as the number of permanent full-time employees. A one unit increase in firm size corresponds to an additional 100 workers as the variable has been scaled for aesthetic purposes. All regressions utilize probability weighting. Regional dummies are Africa, Americas, Asia, Europe, or Oceania. Sector fixed effects are based on two-digit ISIC codes. We drop industries that cannot be estimated with a logit/probit to maintain the same sample across specifications (3), (4), and (5). The country of Malaysia is also dropped due implausible levels of temporary workers (15 firms have greater than 100,000 temporary workers) Standard errors are provided in parenthesis. Asterisks denote significance at the 1% (***) , 5% (**) and 10% (*) levels.