Game Theory

Chapter 12

1 Introduction

- Game theory helps to model ____________ behavior by agents who understand that their actions affect the actions of other agents.
- Game theory applications
  - the study of ____________ (industries containing only a few firms)
  - the study of ____________, e.g., OPEC
  - the study of ____________, e.g., using a common resource such as a fishery
  - the study of ____________ strategies
  - ____________
  - how ____________ work
- A game consists of
  - a set of ____________
    - ⋆ A game with just two players is a ____________ game.
  - a set of ____________ for each player
  - the ____________ to each player for every possible choice of strategies by the players.

1.1 Our First Game

- The players are called A and B.
  - Player A has two actions, called ______________ and ______________
  - Player B has two actions, called ______________ and ______________
- The table showing the payoffs to both players for each of the four possible action combinations is the game's payoff matrix.

		Player B			
		Left		Right	
Player A	Top	3	9	2	8
	Bottom	0	2	1	1

- What do you think would happen if we played this game?
  - Notice that no matter what player B does (L or R), player A is better off choosing ______________.
    - ⋆ This is an example of a ______________ strategy. A dominant strategy is a strategy for a player that is ______________ no matter what the other player does.
    - ⋆ In this example, ______________ is a dominant strategy for A.
    - ⋆ Similarly, ______________ is a dominant strategy for B.

1.2 Our Second Game

		Player B	
		Left	Right
Player A	Top	3 , 9	1 , 8
	Bottom	0 , 0	2 , 1

- What do you think would happen if we played this game?
  - Is (T,R) a likely play?
    - ⋆ If B plays right then A's best reply is ____________ since this improves A's pay-off from ____________.
    - ⋆ (T,R) is ____________ a likely play.
  - Is (B,R) a likely play?
    - ⋆ If B plays right then A's best reply is ____________ and if A plays bottom, B's best reply is ____________
    - ⋆ (B,R) is a ____________ play.
  - Is (B,L) a likely play?
    - ⋆ If A plays bottom then B's best reply is ____________.
    - ⋆ (B,L) is ____________ a likely play.
  - Is (T,L) a likely play?
    - ⋆ If B plays left then A's best reply is ____________ and if A plays top, B's best reply is ____________.
    - ⋆ (T,L) is a ____________ play.
- Nash Equilibrium
  - A play of the game where each strategy is a ____________ reply to the other is a Nash equilibrium (NE).
    - ⋆ Another way to define NE: the set of strategies that are ____________, given what the other player is ____________.
  - Our example has two Nash equilibria: ____________ and ____________.

2 The Prisoner's Dilemma

- To see if ______________-preferred outcomes must be what we see in the play of a game, consider the famous example called the prisoner's dilemma game.
- The game
  - Two players: ______________ and ______________
  - Both have been arrested and taken into questioning. Each have two choices:
    - ⋆ Stay ______________
    - ⋆ ______________
- The Pay-off matrix

		Clyde: Silent		Clyde: Confess	
Bonne	Silent	-5	-5	-30	-1
	Confess	-1	-30	-10	-10

- What plays are we likely to see for this game?
  - If Bonnie stays silent, then Clyde's best response is to ______________ (______________).
  - If Bonnie confesses, Clyde's best response is still to ______________ (______________).
  - ______________ is true for Bonnie.
    - ⋆ Both player's dominant strategy is to ______________.
- The Nash Equilibrium for this game is ______________ even though ______________ would yield better payoffs for both players.
  - The Nash equilibrium here is ______________.
  - The players would be jointly ______________ off each remaining silent. But individual strategies and ______________ lead them each to ______________ since it is a dominant strategy.

3 Repeated Games

- A strategic game is a ____________ game if it is played once in each of a number of periods.

- What strategies are sensible for the players depends greatly on whether the game
  - is repeated over only a ____________ number of periods, or
  - is repeated over an ____________ number of periods.

3.1 Finitely Repeated Games

		Clyde			
		Silent		Confess	
Bonne	Silent	-5	-5	-30	-1
	Confess	-1	-30	-10	-10

- Suppose we have our Bonnie and Clyde Prisoner's dilemma game, but this time it will be repeated for ____________ periods. What is the likely outcome?
  - Suppose the start of period t = 3 has been reached (i.e., the game has already been played twice). Both should choose ____________.
  - Now suppose the start of period t = 2 has been reached. Clyde and Bonnie expect each will choose ____________ next period. Both should choose ____________.
  - At the start of period t = 1 Clyde and Bonnie both expect that each will choose ____________ in each of the next two periods. Both should choose ____________.
- The only ____________ (____________ perfect) NE for this game is where both Clyde and Bonnie choose ____________ in every period. This is true even if the game is repeated for a ____________, but still ____________, number of periods.

3.2 Infinitely Repeated Games

- If the prisoners dilemma game is repeated for an ____________ number of periods then the game has a huge number of credible NE.
  - ____________ forever is one such NE.
  - But ____________ can also be a NE because a player can ____________ the other for not cooperating (i.e., for choosing confess).

4 Who Plays When

- In our previous examples the players chose their strategies ____________.
  - Such games are ____________ games.
- But there are other games in which one player plays ____________ another player.
  - Such games are ____________ games.

  - The player who plays first is the ____________. The player who plays second is the ____________.
- Suppose we had our game from the second example:

		Player B	
		Left	Right
Player A	Top	3, 9	1, 8
	Bottom	0, 0	2, 1

  - But this time the game is played sequentially, with A leading and B following.
    - ⋆ We can rewrite the game in its ____________ form (sometimes called the ____________)

- Solving this kind of game requires a technique known as ____________ induction
  - Even though A goes first, start by figuring out what ______ would do for each possible choice ______ could make.
    - ⋆ If A chooses Top, B would be better off choosing ____________, since ____________.
    - ⋆ If A chooses Bottom, B would be better off choosing ____________, since ____________
  - A knows what B will choose given A's choices, so A will choose ____________ because ____________.

5 Pure Strategies

		Player B	
		Left	Right
Player A	Top	3, 9	1, 8
	Bottom	0, 0	2, 1

- Recall that this game has two Nash Equilibria: (T,L) and (B,R).
  - Player A has been thought of as choosing to play either ____________ or ____________, but no ____________ of both.
  - Similarly, B has been playing either ____________ or ____________, but no ____________ of both.
- In other words, we would say that A is ____________ playing T or B, or T and B are player A's ____________ strategies.
  - We have been thinking of each agent as choosing a strategy ____________ and ____________l. That is, each agent is making one choice and ____________ to it.
  - This means that (T,L) and (B,R) are ____________ strategy Nash equilibria.
    - ⋆ Must every game have at least one pure strategy Nash equilibria?

6 Mixed Strategies

- Do we always want to follow a ____________ strategy?
- Suppose we have the following game:

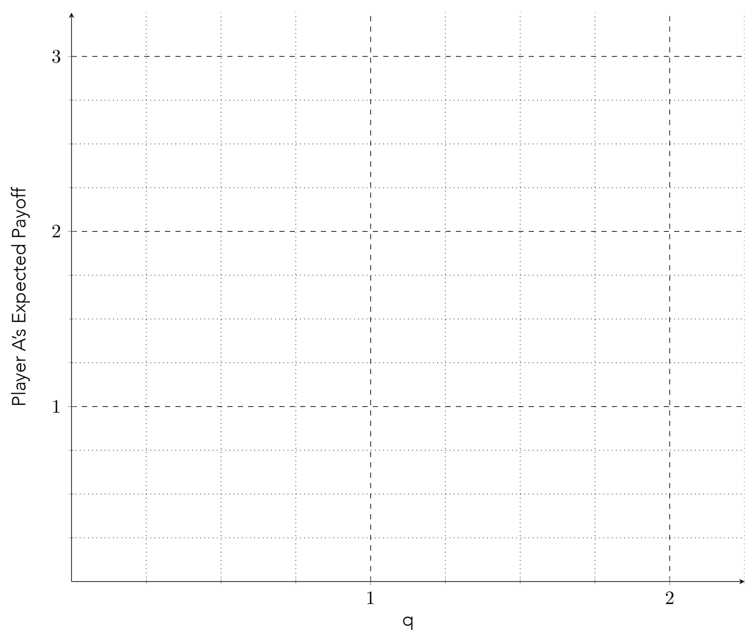
		Player B	
		Left	Right
Player A	Top	1, 2	0, 4
	Bottom	0, 5	3, 2

- Is there a pure strategy Nash Equilibrium? ______
- There is a ____________ strategy Nash equilibrium
  - Instead of playing purely Top or Bottom, player A selects a probability distribution (____________), meaning that with probability ____________ player A will play Top and with probability ____________ will play Bottom.
    - ⋆ Player A is ____________ over the ____________ strategies top and bottom.
    - ⋆ The probability distribution (____________) is a mixed strategy for player A.

- Instead of playing purely Left or Right, player B selects a probability distribution (____________), meaning that with probability ____________ player B will play Left and with probability ____________ will play Right.
  - ⋆ Player B is ____________ over the ____________ strategies left and right.
  - ⋆ The probability distribution (____________) is a mixed strategy for player B.

		Player B	
		Left	Right
Player A	Top	1, 2	0, 4
	Bottom	0, 5	3, 2

- Solving for a mixed strategy Nash Equilibrium
  - Player A knows that player B will play left with probability q and right with probability $1 - q$.
    - ⋆ If player A plays Top, their expected pay-off is:
    - ⋆ If player A plays Bottom, their expected pay-off is:

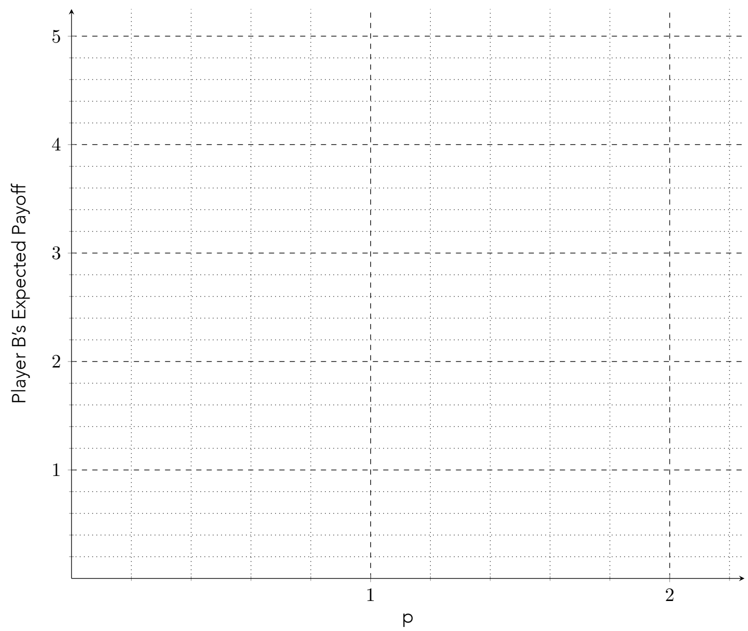


    - ⋆ If ____________, A will choose only top. If ____________, A will choose only bottom.
    - ⋆ If there is a Nash Equilibrium, player A must be ____________ between choosing top or bottom, therefore:

		Player B			
		Left		Right	
Player A	Top	1	2	0	4
	Bottom	0	5	3	2

- Player B knows that play A will play top with probability p and bottom with probability $1 - p$
  - ⋆ If player B plays left, their expected pay-off is:

  - ⋆ If player B plays right, their expected pay-off is:



  - ⋆ If ______________, B will choose only left and if ______________ B will choose only right.
  - ⋆ If there is a Nash equilibrium, then:

		Player B	
		Left	Right
Player A	Top	1, 2	0, 4
	Bottom	0, 5	3, 2

- The Nash Equilibrium for this game is A playing the mixed strategy ____________ and B playing the mixed strategy ____________.
  - ⋆ A's NE expected pay-off is:
  - ⋆ B's NE expected pay-off is:

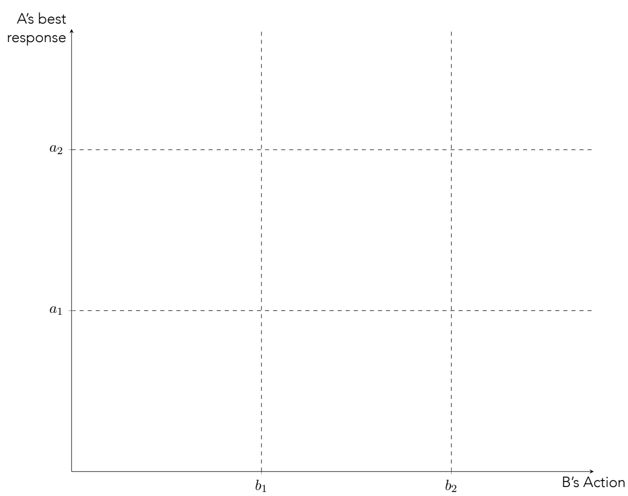
- How Many Nash Equilibria?
  - A game with a ____________ number of players, each with a finite number of ____________ strategies, has at least ____________ Nash equilibrium.
  - So, if the game has ____________ pure strategy Nash equilibrium then it must have at least one ____________ strategy Nash equilibrium.

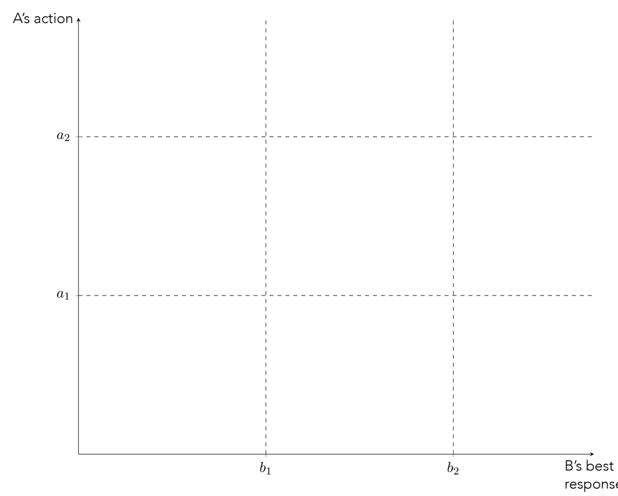
7 Best Response Functions

- In any Nash equilibrium (NE) each player chooses a ____________ response to the choices made by all of the other players. A game may have more than ____________ NE.
  - How can we locate ____________ one of a game's Nash equilibria?
  - If there is more than one NE, can we argue that one is more ____________ to occur than another?
- Think of a 2×2 game; in other words, a game with two players, A and B, each with two actions.
  - A can choose between actions ____________ and ____________.
  - B can choose between actions ____________ and ____________.

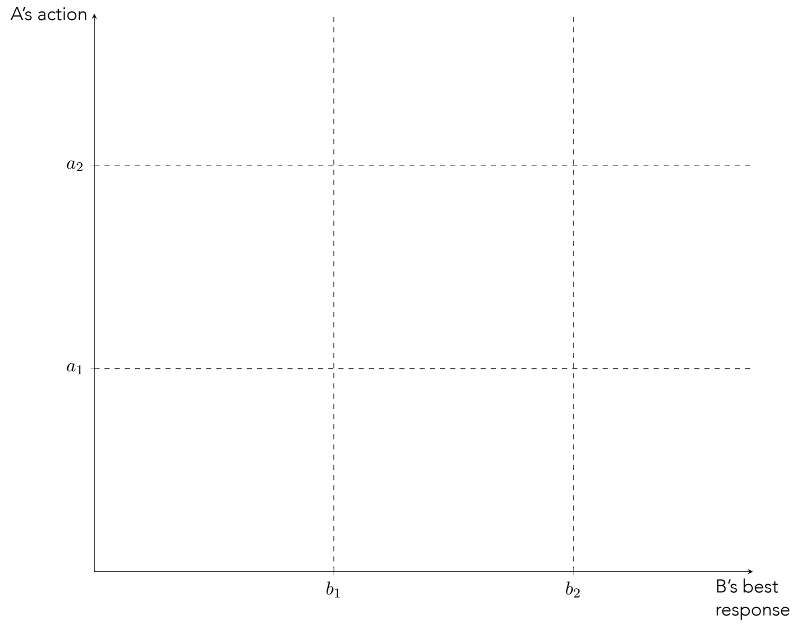
		Player B			
		b_1		b_2	
Player A	a_1	6	4	3	5
	a_2	4	3	5	7

  - We can draw best response curves for A and B:



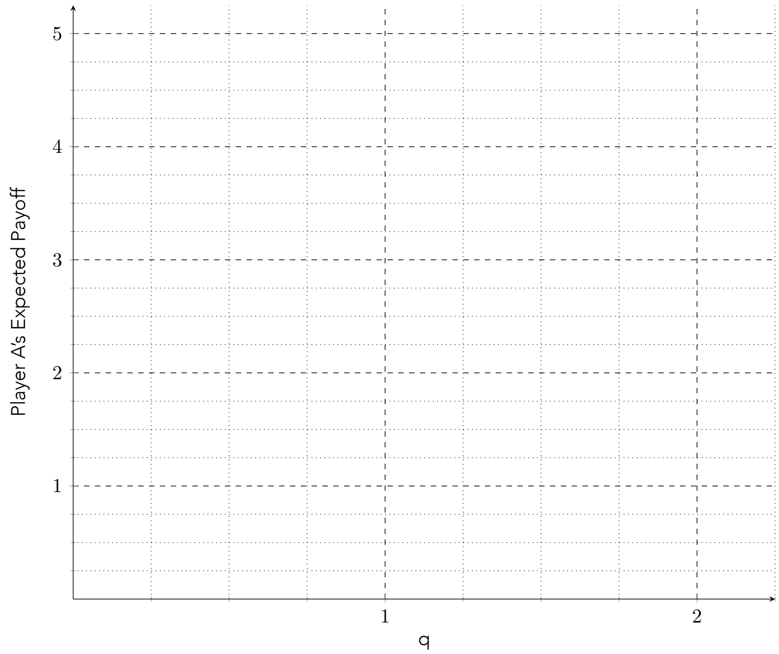


- How can the player's best response curves be used to located the game's Nash Equilibria?
  - ⋆ Put one curve on top of the other.



- What if we allowed the players to ____________ their actions?
  - p is the probability that A chooses action a_1.
  - q is the probability that B chooses action b_1.
- What is the expected value of each action A could take?

  - A is indifferent between the two choices if $EV(a_1) = EV(a_2)$:

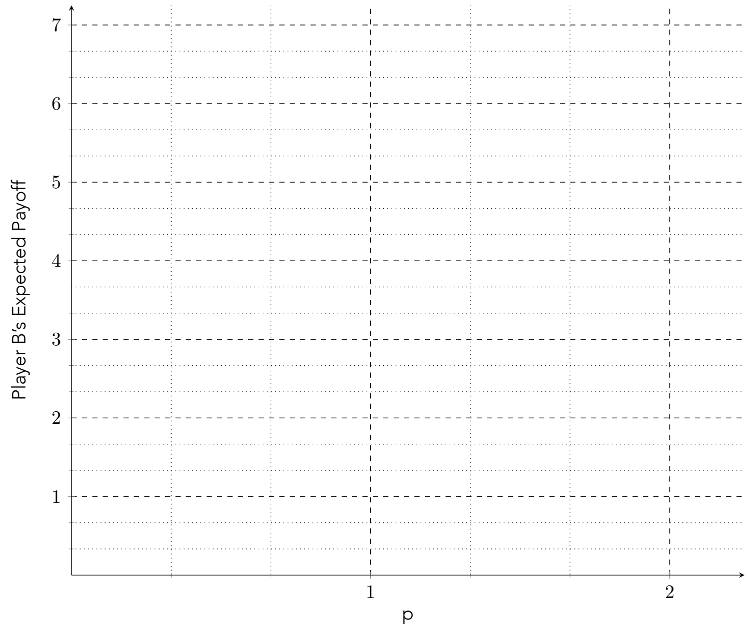


- This implies that A's best response is:

- What is the expected value of each action B could take?

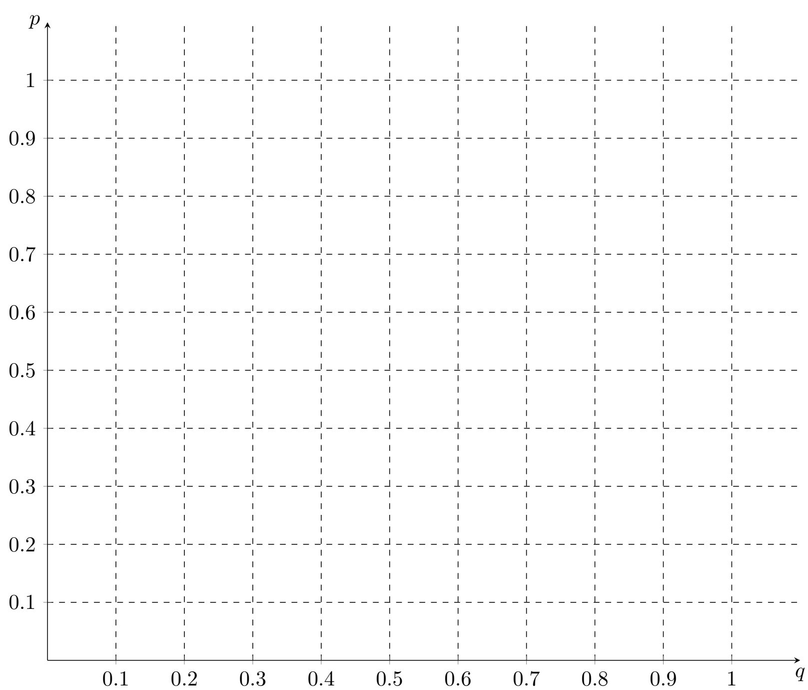
  - B is indifferent between the two choices if $EV(b_1) = EV(b_2)$:

    - ⋆ p cannot be ____________ than 1.



- This implies that B's best response is:

- We can graph out both player's best response functions:

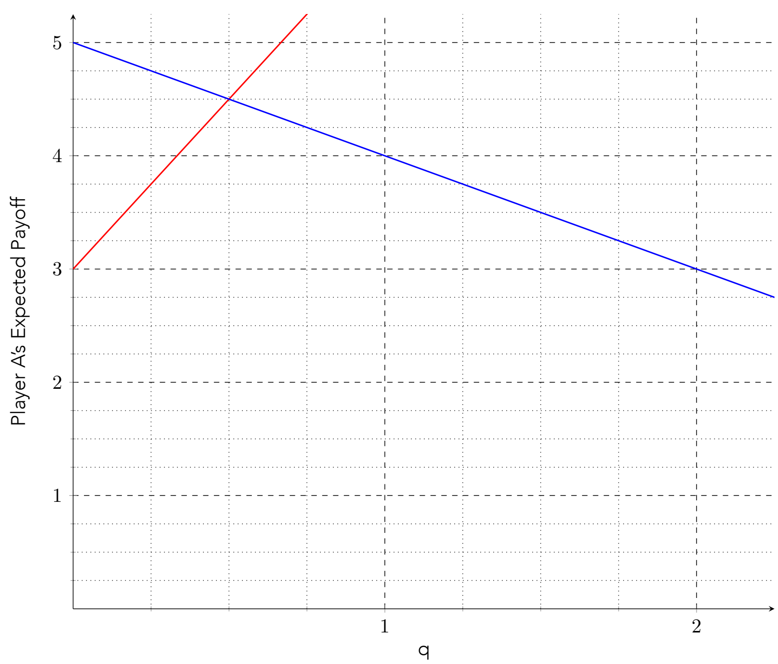


- Suppose we play a slightly different game.

		Player B	
		b_1	b_2
Player A	a_1	6 4	3 1
	a_2	4 3	5 7

- Since the payoffs to A have not changed, the expected values of each action A could take are the same.
  - $EV(a_1) = 6 \times q + 3 \times (1-q) = 3 + 3q$
  - $EV(a_2) = 4 \times q + 5 \times (1-q) = 5 - q$
  - A is indifferent between the two choices if $EV(a_1^A) = EV(a_2^A)$:

$$3 + 3q = 5 - q$$
$$2 = 4q$$
$$q = 1/2$$

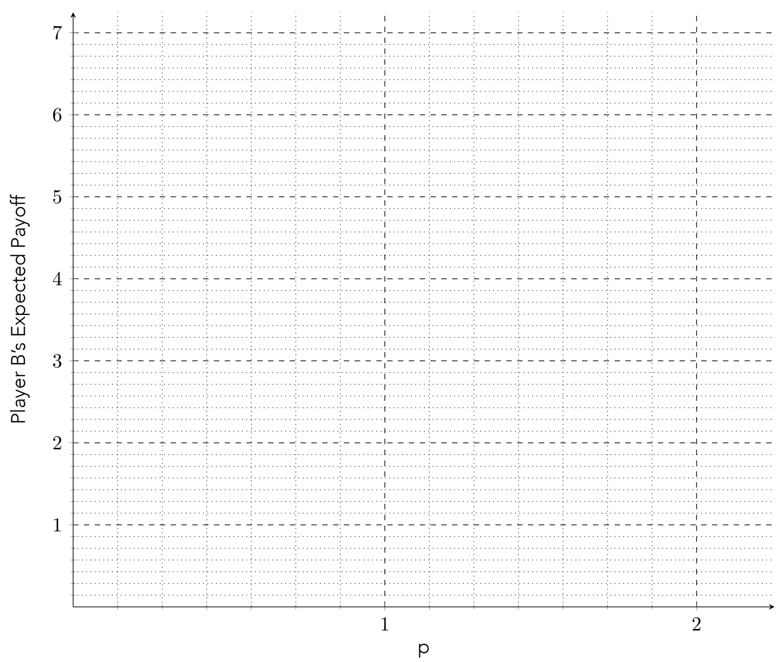


  - The best response function is the same as well:

$$BR_A = \begin{cases} a_1(p=1) & \text{if } q > 1/2 \\ a_2(p=0) & \text{if } q < 1/2 \\ a_1 \text{ or } a_2 (0 \leq p \leq 1) & \text{if } q = 1/2 \end{cases}$$

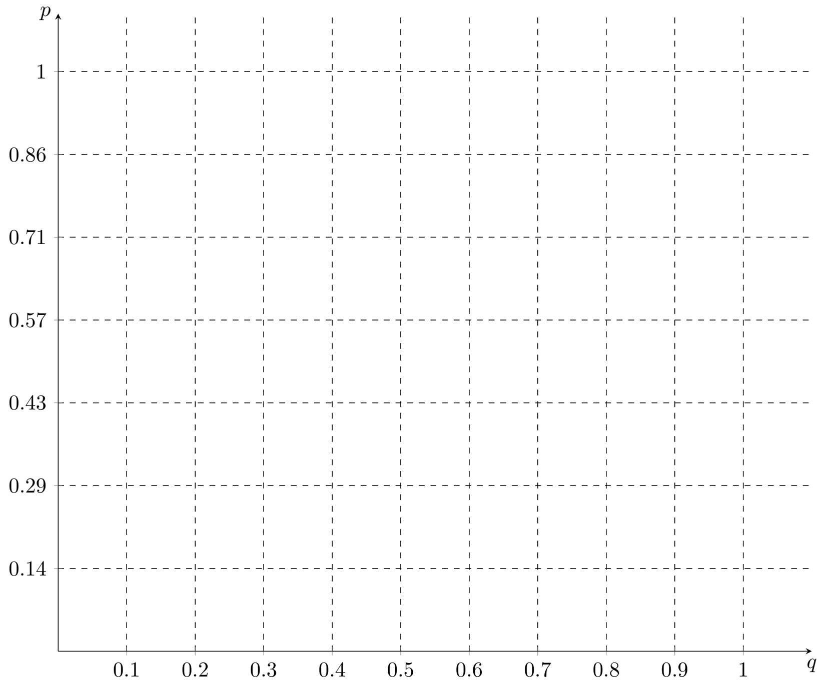
- What is the expected value of each action B could take?

  - B is indifferent between the two choices if $EV(b_1) = EV(b_2)$:



  - This implies that B's best response is:

- We can graph out the player's best response functions:



  - There are three NE for this game: two pure NE and one mixed NE.